International Journal of Innovation Scientific Research and Review
Vol. 05, Issue, 05, pp.4579-4590, May 2023
Available online at http://www.journalijisr.com
SJIF Impact Factor 2023: 6.599

ISSN: 2582-6131

Research Article

THE ADJACENCY SPECTRUM OF THE K(n)(n+1)_α COMPLETE MULTIPARTITION GRAPH

[1,] * Tomas Gonçalves Ferreira, [2]Noor Hidayatand [2]Marjono

[1]Magister Program, Faculty of Mathematics, Brawijaya University Malang, Indonesia.
[2]Faculty of Mathematics and Natural Sciences, Brawijaya University Malang, Indonesia.

Received 17th March 2023; Accepted 18th April 2023; Published online 31st May 2023

ABSTRACT

A graph is a pair of sets (V, E). V is the set of vertices, and E is the set of edges. Graph theory development may be relevant to other areas of mathematics, such as algebra, which are interesting to discuss. A graph can be represented by an adjacency matrix (degree matrix and connectivity of adjacency matrix) whose eigenvalues can be computed. This work aims to find a pattern that will later be used as a theorem for the spectrum of the complete poly-divided graph K(n)(n+1)_α. The results of this study indicate that the general form of the spectrum of a complete multipartition graph K(n)(n+1)α can be used to develop a theorem focused on the spectrum of the dot connected matrix. For further research, it is suggested to continue the research by looking for theorems of various spectrums that can be obtained from the complete graph K(n)(n+1)α to make a complete graph image and find a program.

Keywords: A full multipartition graph's point and side connectedness matrices range.

INTRODUCTION

The capacity to solve difficulties is one of the objectives of learning. One must be able to construct practical mathematical issues to solve these issues. The Königsberg bridge issue, a mathematic problem, introduced graph theory to the world in 1736. This issue arose at Königsberg, now known as Kaliningrad, in the eastern German state of Prussia, where Pregal runs. It surrounds the island of Kneiph of before branching off to form two tributaries (Manuel *et al.,* 2020: Buhaerah *et al.,* 2022). The graph is labelled= $(V(G), E(G))$. Where $V(G)$ is a non-empty infinite set of objects called points, and $E(G)$ is an unordered (possibly empty) set of pairs. Different points of $V(G)$ are called edges. The number of elements of $V(G)$ is called the degree of G, denoted by $p\ (G)$, and the number of factors of $E(G)$ is called the measure of G, denoted by $q(G)$. If the graph we are talking about is just a graph G, then the degree and magnitude of G are written as p and q, respectively. A graph of degree p and size q can be called a graph (p, q) (Brouwer and Haemers, 2012).

The algebraic theory is the branch of mathematics that studies graphs using the algebraic properties of matrices. Let G be a graph of degree $n(n \geq 1)$ and size m, with a set of points $V(G)$ ={$v_1, v_2, \dots, v_n$}. Let A(G) be the point-connected matrix (or adjacency matrix) of graph G. It is a matrix $(n\ x\ n)$ with an entry in the i -th row and j-th column, with values of 1 for direct connections between vi and vj and 0 for indirect references. That is, the adjacency matrix can be composed as $A(G) = [a_{ij}]$, for $1 \leq i, j \leq n$. The adjacency matrix of graph G is a symmetric matrix with elements 0 and 1 and 0 on the main diagonal (Chartrand and Lesniak, 1986).

The degree matrix of matrix G, denoted by $D(G)$, is a diagonal matrix whose elements of the i-th row and j-th column are degrees of v_i with $= 1, 2, 3, \dots, n$. So, the degree matrix of a graph G can be written $D(G) = [d_{ij}]$.,$1 \leq i, j \leq n$. The matrix $L(G) = D(G) - A(G)$ is called the Laplace matrix (Biyikoglu *et al.,* 2009). The matrix $Q(G) = D(G) + A(G)$ is called the Signless-Laplace matrix (BrouwerandHaemers, 2012). The spectrum obtained from the $A(G)$ matrix is called the Adjacency spectrum; from the $L(G)$ matrix, it is called the Laplace spectrum; from the $Q(G)$ matrix, it is called the Signless-Laplace spectrum. The discussion of the $adjacency\ A(G)$, the Laplace matrix $L(G)$, and the Signless-Laplace matrix $Q(G)$, from the graph G can be related to the concept of eigenvalues and eigenvectors on the topic of linear algebra that produces the concept of the spectrum. Suppose that $\lambda_0,\ \lambda_1, \dots, \lambda_{\delta-1}$ with $\lambda_0 \leq \lambda_1, \dots, \lambda_{\delta-1}$ is a different eigenvalue from the matrix of a graph, and suppose $m(\lambda_0), m(\lambda_1), \dots, m(\lambda_{\delta-1})$ is the multiplicity of its eigenvalues λ_i. An ordered matrix (2 x n) containing $\lambda_0,\ \lambda_1, \dots, \lambda_{\delta-1}$ in the first line and $m(\lambda_0), m(\lambda_1), \dots, m(\lambda_{\delta-1})$ in the second line is called the graph spectrum G and is annotated with $Spec(G)$.So, the spectrum of graph G can be composed with

$$\text{Spec}(G) = \begin{bmatrix} \lambda_0 & \lambda_1 & \cdots & \lambda_{\delta-1} \\ m(\lambda_0) & m(\lambda_1) & \cdots & m(\lambda_{\delta-1}) \end{bmatrix}$$

Adjacency spectrum is the name given to the spectrum derived from matrix A(G). There are several studies, including the adjacency spectrum study, which includes [1] Glen, (2018), who identify the adjacency spectrum of two novel operators. Chartrand *et al.,* (2016) deduced the complement dihedral group's spectrum adjacency from the sub graph group [2]. This article will discuss how to determine the general form-spectrum adjacency of complete multipartitiongraphs $K(n)(n+1)_\alpha$. Spectrum determination begins with representing the graph in a matrix, then determining the eigenvalues and vector eigen-the relationship between eigenvalues and thedimensions of eigenvector spaces, after which in a graph spectrum matrix.

***Corresponding Author: Tomas Gonçalves Ferreira,**
1Magister Program, Faculty of Mathematics, Brawijaya University Malang, Indonesia.

GRAPH THEORY

Graph theory is an old subject but has many applications [2]—discrete items and their connections as displayed using graphs. The visual representation of a graph is to state an object expressed as (noktah), sphere, or point, whereas a line expresses the relationship between objects. The definition of graph G in mathematics is:The pair of sets (V, E) where V is the infinitely empty set of elements called points $(vertex)$ and E is the finite set (can be empty) of point pairs in $Vx\ V$ called sides $(edges)$. $V$is called the set of points, and E is called the set of sides of the graph G. Each side in V condenses two points from V. Other terms of points are $(nodes)$, and other terms of sides are $(lines)$.

Definition of degrees and incidence

a. Suppose $G = (V, E)$ a graph if two points u and v in G are connected by the sides $e = uv$, then points u and v is said to be close together $(adjacency)$. If side $e = uv,$ point u and side e (also $v$and e) are said $(incident)$.

b. If the two sides of e_1 and e_2 in G are next to one another and share a point of communion, they are said to be $adjacency$.

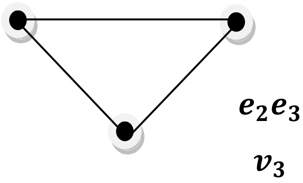


Figure 2.1 Two adjacent points

From Figure 2.1, e_1 and e_2 are adjacent because they are both side by side with the v_1 point. Likewise $e_1$and e_3; e_2 and e_3.

c. A graph labeled with every point as a single one is called a labeled Graph.

Example 2.1 Graph G in Figure 2.2, the dots are labeled $v_1, v_2, v_3, v_4, v_5, v_6$, and v_7.

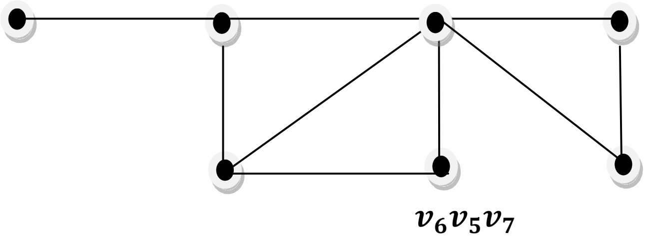


Figure 2.2 Graph labeled G

d. The $(order)$ of graph G is the number of points in graph G. The $(size)$ of graph G is the size of the inner side of graph G.Figure 2.2 Shows a G graph with $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E = \{v_1v_2, v_2v_3, v_2v_4. v_3v_4, v_3v_5, v_3v_6, v_4v_5, v_6v_7\}$. Since graph G has seven points and nine sides, order G is seven, and the size is nine.

e. The degree $(degree\ valency)$ of the point v in G is the number of sides side by side with v.

Example 2.1 (Chartrand and Lesniak, 1986).

A one-order graph with an empty set of sides is said to be trivial. A graph that orders more than one is said to be nontrivial.

Tracks and Circuits.

In the definition of connected graphs, G. There are several terms for graphs called roads $(walk)$, $(path)$, $(trail)$, and $(circuit)$.

a. The $u - v(walk)$ in graph G is a line of dots in G that starts from point u and ends at point v so that the dots dotted in the row are adjacent $(adjacency)$, written.

$$W : u = v_1, v_2, \ldots, v_k = v.$$

where$k \geq 0$; v_i and v_{i+1} adjacency for $i = 0,1, \ldots, k - 1$. If $u = v$, then W is a closed road, while if $u \neq vW$ is an open road. The number of sides that appear in a $walk$ (including sides that appear more than one) is an open road.

Example 2.2 Figure 2.3 shows that the$v_1 - v_8$track:$v_1 v_2 v_3 v_6 v_2 v_3 \boldsymbol{v_8}$is an open $walk$ with a length of 6

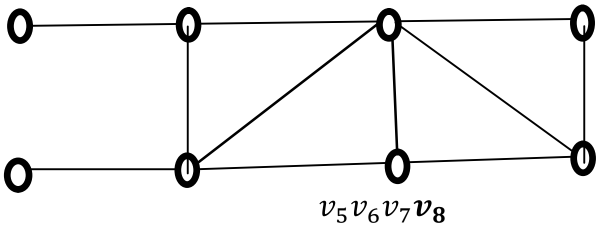


Figure 2.3$walk\ W :\ v_1 v_2 v_3 v_6 v_2 v_3 \boldsymbol{v_8}$

b. The$u - v(path)$in graph$G$ is a $u - v\ walk$ where all points (except closed $walk$) intersect.
In Figure 2.3, the path v1−v8 p: v1v2v3v6v2v3v8 has a path length of 4. The length of the shortest path from u to v is called the distance from u to v, denoted $d(u, v)$ the shortest $path$ from u to v is called $geodesic$ and $u$to v.

c. A $(trail)$ is a $walk$ where all sides are different.
So, every trail $(path)$ is a $trail$, but every trail is not necessarily a $path$.

Example 2.3 in Figure 2.3

$W : v_1v_2v_3v_6v_7v_3v_8$is a $(trail)$ but not a $(path)$
$W : v_1v_2v_3v_6v_7v_8$is a $(path)$ but not a $(trail)$
A closed $Trail$ that contains all points and sides is called a $trail$.

d. A circuit is a closed trail whose point does not appear more than once, except for the starting and endpoints.

Another name for a circuit is a $(cycle)$ or circular route. An even sicle is a sicle that is the same length as another sicle, whereas an odd sicle is the opposite. A sicle of length k is represented by the notation Cycle$-k$.

Example 2.4 In graph G in Figure 2.3

C: $v_2v_3v_8v_7v_7v_6v_2$is a sicle

Definition of Complete Multipartition Graph

A complete graph G m-partition is an m-partition graph with a set of partitions $V_1, V_2, \ldots, V_m$ has the additional condition that if $u \in V_i$ and $v \in V_j, i \neq j$,then $uv \in E(G)$. If | $|V_i| = \alpha_i$, then this graph is annotated $K(\alpha_1, \alpha_2, \ldots \alpha_n)$, (Chartrand and Lesniak, 1986).

In this study, the complete multipartition graph $K(n)(n+1)_\alpha$ states a full multipartition graph with one partition containing as many as $\boldsymbol{n}$ points and as many as α other partitions having as many as $(n+1)$ issues. Some properties of the $adjacency$ spectrum (Manuel *et al.*, 2020: Glen *et al.*, 2018) determine the $adjacency$ spectrum of the $S_n$star graph is:

Graph The adjacency spectrum of the star graph $\mathbf{S_n}$.

Crown graph S_n^0, ladder graph L_n. The $adjacency$ spectrum of the S_n star graph is a complete bipartite graph $K_{m,n}$ where M is the set of singletons. On the stellar graph, S_n consists of 1 $n-1$ point called the center point and $n-1$ point 1 called the leaf. Theorem seven regarding the spectrum S_n.

Theorem 1. For example, S_n is a star graph, then the spectrum of the S_n star graph is

$$Spec(S_n) = \begin{bmatrix} \sqrt{n} & 0 & -\sqrt{n} \\ 1 & n-1 & 1 \end{bmatrix}$$

Proof: Known $A(S_n)$ is the $adjacency$ matrix of the star graph S_n is

$$A(S_n) = \begin{bmatrix} 0 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Look for the value of **λ** such that SPL $(A - \lambda I)x = 0$ has a non-trivial solution, namely the importance of **λ** that satisfies $det(A - \lambda I)$ as follows:

$$det(A - \lambda I) = 0$$

$$\Leftrightarrow det \begin{bmatrix} -\lambda & 1 & 1 & 1 & \cdots & 1 \\ 1 & -\lambda & 0 & 0 & \cdots & 0 \\ 1 & 0 & -\lambda & 0 & \cdots & 0 \\ 1 & 0 & 0 & -\lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & 0 & -\lambda \end{bmatrix} = 0$$

$\Leftrightarrow (-1)^{n+1}(\lambda)^{n-1}(\lambda^2 - n) = 0,$

Obtained eigenvalues

$\lambda_1 = \sqrt{n}$,$m_\alpha(\lambda_1) = 1, \lambda_2 = 0, m_\alpha(\lambda_2) = n-1, \lambda_3 = -\sqrt{n}. m_\alpha(\lambda_3) = -\sqrt{n}, m_\alpha(\lambda_3) = 1.$

The $adjacency$ spectrum of the star graph S_n is as follows:

$$Spec\ (S_n) = \begin{bmatrix} -\sqrt{n} & 0 & \sqrt{n} \\ 1 & n-1 & 1 \end{bmatrix}$$

Adjacency spectrum of the crown graph $\mathbf{S_n^0}$

A crown graph S_n^0 is a graph that has several points of $2n$ and several sides $n(n-1)$. The $adjacency$ matrix of the S_n^0 crown graph can be partitioned into two submatrices: the complete matrix K_n and the zero matrices. Thus, determining the spectrum of the crown graph will be easier. To be clear, given Theorem 2 as follows:

Theorem 2. If S_n^0 is a crown graph, then the spectrum of the crown graph S_n^0 is

$$Spec(S_n^0) = \begin{bmatrix} n-1 & 1 & -1 & 1-n \\ 1 & n-1 & n-1 & 1 \end{bmatrix}$$

Proof: Known $A(S_n^0)$ is the $adjacency$ matrix of the crown graph, then

$$A(S_n^0) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 1 & \dots & 0 \\ 0 & 1 & \dots & 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} = \left[\begin{array}{cc} 0 & A[K_n] \\ \hline A[K_n] & 0 \end{array}\right]$$

The $adjacency$ matrix of the crown graph can be partitioned into a zero matrix and the $adjacency$ matrix of the complete graph $A(K_n)$. Since the entry $A(S_n^0)$ is composed of $A(K_n)$, then the eigenvalues of $A(S_n^0)$ can be formed from $A(K_n)$. Jones [2] explains that the $adjacency$ spectrum of the complete graph $A(K_n)$is $A(K_n) = \begin{bmatrix} n-1 & -1 \\ 1 & n-1 \end{bmatrix}$.

Suppose that λ is the eigenvalue of the adjacency matrix $A(K_n)$, λ^* is the eigenvalue of the adjacency matrix $A(S_n^0)$, and v_λ is the eigenvector corresponding to $\boldsymbol{\lambda}$. Then, it is provided with that$x = \begin{bmatrix} v_\lambda \\ v_\lambda \end{bmatrix}$, $y = \begin{bmatrix} v_\lambda \\ -v_\lambda \end{bmatrix}$then for x obtained:

$$A(S_n^0)x = \begin{bmatrix} 0 & A(K_n) \\ A(K_n) & 0 \end{bmatrix}\begin{bmatrix} v_\lambda \\ v_\lambda \end{bmatrix} = \begin{bmatrix} A(K_n)v_\lambda \\ A(K_n)v_\lambda \end{bmatrix} = \begin{bmatrix} \boldsymbol{\lambda v_\lambda} \\ \boldsymbol{\lambda v_\lambda} \end{bmatrix} = \boldsymbol{\lambda}\begin{bmatrix} \boldsymbol{v_\lambda} \\ \boldsymbol{v_\lambda} \end{bmatrix} = \boldsymbol{\lambda x}$$

Obtained which is the eigenvalue of $\mathrm{A}(\mathrm{S_n^0})$ so that $\lambda^* = \lambda = 1$ with an algebraic and geometric multiplicity of $\mathrm{n} - 1$. Whereas $\lambda^* = \boldsymbol{\lambda} = \boldsymbol{n} - \mathbf{1}$ with an algebraic and geometric variety of 1. Then for y, it is obtained:

$$A(S_n^0)y = \begin{bmatrix} 0 & A(K_n) \\ A(K_n) & 0 \end{bmatrix}\begin{bmatrix} v_\lambda \\ -v_\lambda \end{bmatrix} = \begin{bmatrix} A(K_n)(-v_\lambda) \\ A(K_n)v_\lambda \end{bmatrix} = \begin{bmatrix} A(-v_\lambda) \\ \lambda v_\lambda \end{bmatrix} = \begin{bmatrix} -\lambda v_\lambda \\ -\lambda(-v_\lambda) \end{bmatrix} = \boldsymbol{\lambda}\begin{bmatrix} \boldsymbol{v_\lambda} \\ \boldsymbol{-v_\lambda} \end{bmatrix} = \boldsymbol{-\lambda y}$$

Obtained $\boldsymbol{-\lambda}$ is the eigenvalue of $A(S_n^0)$ so that $\lambda^* = \boldsymbol{-\lambda} = -(\mathbf{-1}) = \mathbf{1}$ with an algebraic and geometric multiplicity of $n-1$. Whereas $\lambda^* = \boldsymbol{-\lambda} = -(\boldsymbol{n}-\mathbf{1}) = \mathbf{1} - \boldsymbol{n}$ with an algebraic and geometric variety of 1. Thus, it is proven that:

$$Spec(S_n^0) = \begin{bmatrix} n-1 & 1 & -1 & 1-n \\ 1 & n-1 & n-1 & 1 \end{bmatrix}.$$

Spektrum adjacency darigraftangga$\mathbf{L_n}$

The L_n ladder graph is a simple directionless graph derived from Cartesian results of the P_n trajectory graph and the P_2 trajectory graph. The number of points on the L_n ladder graph is $2n$, and the number of sides is $3n-2$. At the time of determining the spectrum of the graph, of course, what is needed is the eigenvalue of $A(G)$. In some graphs G, the eigenvalue of $A(G)$ is only sometimes determined easily, so a special tool is used to determine the pattern of its $adjacency$ spectrum; in this case, the device used is the $Chebyshev$ polynomial. The $Chebyshev$ polynomial contains the functions of sine and cosine (Harary and Schwenk, 1974). An example of applying the $Chebyshev$ polynomial is the determination of the $adjacency$ spectrum of the P_n trajectory graph. The adjacency spectrum of the P_n trajectory graph is (Biggs, 1993):

$$\begin{bmatrix} 2\,cos\left(\frac{k\pi}{n+1}\right) & \cdots & 2\cos\left(\frac{n\pi}{n+1}\right) \\ 1 & \cdots & 1 \end{bmatrix}, k = 1,2,\dots,n$$

Similar to the crown graph, the $adjacency$ matrix of the ladder graph can L_n be partitioned into two submatrices, namely the identity matrix and the P_n trajectory matrix. For more details is given Theorem 3 as follows:

Theorem 3. For example, L_n is the graph of the ladder order n, then the graph spectrum of the L_n ladder is:

$$Spec\ (L_n) = \begin{bmatrix} 1 - 2\cos\left(\frac{n\pi}{n+1}\right) & \cdots & -1 + 2\cos\left(\frac{n\pi}{n+1}\right) \\ 1 & \cdots & 1 \end{bmatrix}$$

Proof: Known $A(L_n)$ is the $adjacency$ matrix of the ladder graph, then

$$A(L_n) = \begin{bmatrix} 0 & 1 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} A(P_n) & 1 \\ 1 & A(P_n) \end{bmatrix}$$

It is known that the $adjacency$ matrix of a ladder graph can be partitioned into an identity matrix and an $adjacency$ matrix of path graph $A(P_n)$. Let λ be the eigenvalues of the $adjacency$ matrix (P_n). λ^* are the eigenvalues of the $adjacency$ matrix $A(L_n)$ and v_λ be the corresponding eigenvectors. Suppose $x = \begin{bmatrix} v_\lambda \\ v_\lambda \end{bmatrix}$ $y = \begin{bmatrix} v_\lambda \\ -v_\lambda \end{bmatrix}$, then for x will be obtained

$$A(L_n)x = \begin{bmatrix} A(P_n) & I_n \\ I_n & A(P_n) \end{bmatrix} \begin{bmatrix} v_\lambda \\ v_\lambda \end{bmatrix} = \begin{bmatrix} A(P_n)v_\lambda + I_n v_\lambda \\ A(P_n)v_\lambda + I_n v_\lambda \end{bmatrix} = \begin{bmatrix} \lambda v_\lambda + v_\lambda \\ \lambda v_\lambda + v_\lambda \end{bmatrix} = \begin{bmatrix} \lambda v_\lambda + v_\lambda \\ (\lambda + 1) + v_\lambda \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda + 1)v_\lambda \\ (\lambda + 1)v_\lambda \end{bmatrix} = (\lambda + 1) \begin{bmatrix} v_\lambda \\ v_\lambda \end{bmatrix}$$

Obtained $\lambda + 1$, which is the eigenvalue of $A(I_n)$ so that $\lambda^* = (\lambda + 1) = 2\, cos\left(\frac{n\pi}{n+1}\right) + 1$ with algebraic and geometric multiplicity is 1. Then, vector y will be obtained

$$A(L_n)y = \begin{bmatrix} A(P_n) & 1 \\ 1 & A(P_n) \end{bmatrix} \begin{bmatrix} v_\lambda \\ -v_\lambda \end{bmatrix} = \begin{bmatrix} \left(A(P_n)v_\lambda + I_n(-v_\lambda)\right) \\ \left(A(P_n)(-v_\lambda) + I_n v_\lambda\right) \end{bmatrix} = \begin{bmatrix} \lambda v_\lambda - v_\lambda \\ \lambda(-v_\lambda) + v_\lambda \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda - 1)v_\lambda \\ (\lambda - 1)(-v_\lambda) \end{bmatrix} = (\lambda - 1) \begin{bmatrix} v_\lambda \\ -v_\lambda \end{bmatrix}$$

It is obtained that $\lambda - 1$ is the eigenvalue of $A(I_n)$ so that $\lambda^* = (\lambda - 1) = -1 + 2\, cos\left(\frac{n\pi}{n+1}\right)$ with algebraic and geometric multiplicity is 1. So, it is proved that:

$$Spec(L_n) = \begin{bmatrix} 1 + 2\cos\left(\frac{n\pi}{n+1}\right) & \cdots & -1 + 2\, cos\left(\frac{n\pi}{n+1}\right) \\ 1 & \cdots & 1 \end{bmatrix}.$$

RESEARCH METHODS

This study employed a library research approach, a literature review, to gather the facts, figures, and artefacts needed to examine the issue at hand. A literature study presents scientifically sound reasons to explain the outcomes of thinking about a problem or subject covered in the research's literature review. A computer application called Maple 12 is used for matrix computations utilizing symbols and numerical inputs throughout computing patterns.

The phases of this research are:

a. Describes the complete multipartition graph pattern $K(n)(n+1)_\alpha$
b. Formulate the spectral pattern of the point-connected matrix of a complete multipartition graph $K(n)(n+1)_\alpha$
c. The resulting conjecture is then proved by formulating the belief as a theorem supplemented by proof.
d. Provide the research's findings and conclusions.

RESULTS AND DISCUSSION

Spectrum of Complete MultipartitionGraphs $\mathbf{K(1)(2)_\alpha}$, $\mathbf{\alpha \geq 2}$.

This section will describe determining the spectrum adjacency of various complete multipartition graphs $K(1)(2)_\alpha$.For $\alpha = 1,2,3,\dots,k$, the explanation for $\alpha = 2,3,4$ will be described as follows. If $\alpha = 2$, then the graph is shown in figure 4.1 below.

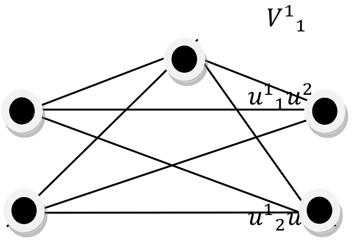


Figure 4.1 Complete MultipartitionGraph$K(1)(2)_2$

Furthermore, the connectedness matrix of the complete multipartition graph $K(1)(2)_2$ is

$$:v^1{}_1 \quad u^1{}_1 \quad u^1{}_2 \quad u^2{}_1 \quad u^2{}_2$$

$$A = \begin{array}{c} {v^1}_1 \\ {u^1}_1 \\ {u^1}_2 \\ {u^2}_1 \\ {u^2}_2 \end{array} \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

By eliminating the matrix using the Gauss Elimination method, the characteristic polynomial of $(A(K(1)(2)_2) - \lambda I)$ is $det(A(K(1)(2)_2) - \lambda I)$, is the result multiplying all the elements of the main diagonal of the upper triangular matrix, so we get: $det(A(K(1)(2)_2) - \lambda I) = (1)(\lambda)^2(2+\lambda)(-\lambda^2 + 2\lambda + 4)$ because $det(A(K(1)(2)_2) - \lambda I) = 0$ then $det(A(K(1)(2)_2) - \lambda I) = (1)(\lambda)^2(2+\lambda)(-\lambda^2 + 2\lambda + 4) = 0$ and the eigenvalues are obtained $\lambda_1 = 1 + \sqrt{5}, \lambda_2 = 1 - \sqrt{5, }\lambda_3 = 0, \lambda_4 = -2$. Next, we will look for the basis of the eigenvector space $\lambda_1 = 1 + \sqrt{5}$. By substituting these values into $det\ (A(K(1)(2)_2) - \lambda I)$ we get the following:

$$\begin{bmatrix} -\lambda_1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_1 & 0 & 1 & 1 \\ 1 & 0 & -\lambda_1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_1 & 0 \\ 1 & 1 & 1 & 0 & -\lambda_1 \end{bmatrix}$$

$$= \begin{bmatrix} -(1+\sqrt{5}) & 1 & 1 & 1 & 1 \\ 1 & -(1+\sqrt{5}) & 0 & 1 & 1 \\ 1 & 0 & -(1+\sqrt{5}) & 1 & 1 \\ 1 & 1 & 1 & -(1+\sqrt{5}) & 0 \\ 1 & 1 & 1 & 0 & -(1+\sqrt{5}) \end{bmatrix}$$

By using the Gauss-Jordan technique to remove the matrix with the Maple 12 program, the outcomes that followed were as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \dfrac{+(20+9\sqrt{5})}{(2\sqrt{5}+5)(7+3\sqrt{5})} \\ 0 & 1 & 0 & 0 & \dfrac{65+29\sqrt{5}}{(2\sqrt{5}+5)(7+3\sqrt{5})} \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above results show that the number of bases of the eigenvector space corresponding to $\lambda_1 = 1 + \sqrt{5}$ is 1. Next, we will look for the basis of the eigenvector space $\lambda_2 = 1 - \sqrt{5}$. By substituting these eigenvalues into $\det\ (A(K(1)(2)_2) - \lambda I)$ it is obtained as follows:

$$\begin{bmatrix} -\lambda_2 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_2 & 0 & 1 & 1 \\ 1 & 0 & -\lambda_2 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_2 & 0 \\ 1 & 1 & 1 & 0 & -\lambda_2 \end{bmatrix} = \begin{bmatrix} 1-\sqrt{5} & 1 & 1 & 1 & 1 \\ 1 & 1-\sqrt{5} & 0 & 1 & 1 \\ 1 & 0 & 1-\sqrt{5} & 1 & 1 \\ 1 & 1 & 1 & 1-\sqrt{5} & 0 \\ 1 & 1 & 1 & 0 & 1-\sqrt{5} \end{bmatrix}$$

Eliminating the matrix using the Gauss-Jordan method, carried out with Maple 12 software, gives the following results:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \dfrac{4(20+9\sqrt{5})}{(2\sqrt{5}-5)(-7+3\sqrt{5})} \\ 0 & 1 & 0 & 0 & \dfrac{-65+29\sqrt{5}}{(2\sqrt{5}-5)(-7+3\sqrt{5})} \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above results show that the number of bases of the eigenvector space corresponding to $\lambda_2 = 1 - 5$ a. Next, we will look for the basis of the eigenvector space $\lambda_3 = 0$.By substituting these values into $det(A(K(1)(2)_\alpha) - \lambda I)$ the following is obtained:

$$\begin{bmatrix} -\lambda_3 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_3 & 0 & 1 & 1 \\ 1 & 0 & -\lambda_3 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_3 & 0 \\ 1 & 1 & 1 & 0 & -\lambda_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Eliminating the matrix using the Gauss-Jordan method, carried out with Maple 12 software, gives the following results:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the results above, that is obvious that the number of bases of the eigenvector space corresponding to $\lambda_3 = 0$ is 2. Next, we will look for the basis of the eigenvector space $\lambda_4 = -2$. By substituting these values into $det(A(K(1)(2)_2) - \lambda I)$ we get the following:

$$\begin{bmatrix} -\lambda_4 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_4 & 0 & 1 & 1 \\ 1 & 0 & -\lambda_4 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_4 & 0 \\ 1 & 1 & 1 & 0 & -\lambda_4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

By eliminating the matrix by the Gauss Jordan method carried out with the help of Maple 12 software, obtained as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The results above show that the number of the basis of the eigenvector space corresponding to $\lambda_4 = -2$ is 2. Based on these results, the spectrum of the complete multipartition graph $K(1)(2)_2$ is obtained

$$Spec\, A(K(1)(2)_2) = \begin{bmatrix} 1+\sqrt{5} & 1-\sqrt{5} & 0 & -2 \\ 1 & 1 & 2 & 1 \end{bmatrix}.$$

Now will be drawn the complete multidirectional graph spectrum $K(1)(2)_3$

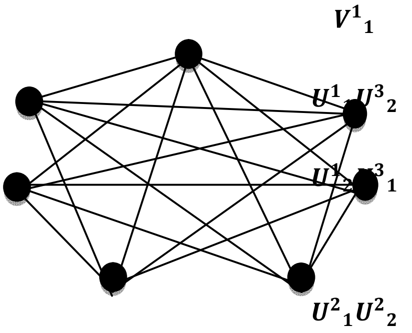


Figure 4.2 Complete MultipartitionGraph$K(1)(2)_3$

Based on Figure 4.2 which is a complete multipartitiongraph $\mathrm{K}_{(1)(2)_3}$, it produces the dot-connected matrix as follows:

$$A(K(1)(2))_3 = \begin{array}{c|ccccccc} & V^1{}_1 & U^1{}_1 & U^1{}_2 & U^2{}_1 & U^2{}_2 & U^3{}_1 & U^3{}_2 \\ \hline V^1{}_1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ U^1{}_1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ U^1{}_2 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ U^2{}_1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ U^2{}_2 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ U^3{}_1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ U^3{}_2 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{array}$$

After getting the dot-connected matrix form, we will look for the eigenvalues and eigenvectors of the matrix $\det(A(K(1)(2)_3) - \lambda I) = 0$ obtained:

$$det \begin{bmatrix} -\lambda & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -\lambda & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -\lambda & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -\lambda \end{bmatrix} = 0$$

It will be acquired using the Gauss Elimination technique to remove the matrix

$$\begin{bmatrix} 1 & -\lambda & 0 & 1 & 1 & 1 & 1 \\ 0 & \lambda & -\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 2+\lambda & -\lambda-1 & -1 & 0 & 0 \\ 0 & 0 & 0 & \lambda & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+\lambda & -\lambda-1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \lambda & -\lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda^2+4\lambda+6 \end{bmatrix}$$

The characteristic polynomial of $(A(K(1)(2)_3)-\lambda I)$ is $det(A(K(1)(2)_3)-\lambda I)$ is the product of all the elements of the main diagonal of the upper triangular matrix, so we get: $det(A(K(1)(2)_3)-\lambda I)=(1)(\lambda)^3(2+\lambda)(-\lambda^2+4\lambda+6)$,, because $det(A(K(1)(2)_3)-\lambda I)=0$,, then $det(A(K(1)(2)_3)-\lambda I)=(1)(\lambda)^3(2+\lambda)(-\lambda^2+4\lambda+6)=0$. And obtained the eigenvalues $\lambda_1=2+\sqrt{10}$, $\lambda_2=2-\sqrt{10}, \lambda_3=0, \lambda_4=-2$. Next, we will look for the basis of the eigenvector space $\lambda_1=2+\sqrt{10}$. Substituting this value into $\det(A(K(1)(2)_3)-\lambda I)$ is obtained as follows:

$$\begin{bmatrix} -\lambda_1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -\lambda_1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -\lambda_1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -\lambda_1 \end{bmatrix} \begin{bmatrix} -(2+\sqrt{10}) & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -(2+\sqrt{10}) & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -(2+\sqrt{10}) & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -(2+\sqrt{10}) & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -(2+\sqrt{10}) & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -(2+\sqrt{10}) & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -(2+\sqrt{10}) \end{bmatrix}$$

It is derived as follows by using the Gauss-Jordan technique to eliminate the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \dfrac{2(14+5\sqrt{10})}{(4+\sqrt{10})} \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The results above show that the number of bases of the eigenvector space corresponding to $\lambda_1 = 2+\sqrt{10}$ is 1. Next, we will look for the basis of the eigenvector space $\lambda_2 = 2-\sqrt{10}$.By substituting these values into $\det(A(K(1)(2)_3)-\lambda I)$ we get:

$$\begin{bmatrix} -\lambda_2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_2 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -\lambda_2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -\lambda_2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -\lambda_2 \end{bmatrix}$$

$$= \begin{bmatrix} -(2-\sqrt{10}) & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -(2-\sqrt{10}) & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -(2-\sqrt{10}) & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -(2-\sqrt{10}) & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -(2-\sqrt{10}) & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -(2-\sqrt{10}) & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -(2-\sqrt{10}) \end{bmatrix}$$

By eliminating the matrix by the Gauss-Jordan method, obtained:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \dfrac{2(-14+5\sqrt{10})}{(-4+\sqrt{10})^2} \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The results above show that the number of bases of the eigenvector space corresponding to $\lambda_2 = 2-\sqrt{10}$ is 1. Next, we will look for the basis of the eigenvector space $\lambda_3 = 0$. By substituting these values into $det(A(K(1)(2)_3)-\lambda I)$ we get:

$$\begin{bmatrix} -\lambda_3 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_3 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -\lambda_3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_3 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -\lambda_3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & -\lambda_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Eliminating the matrix by the Gauss-Jordan method is obtained as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The results above show that the number of bases of the eigenvector space corresponding to $\lambda_3 = 0$ is 3. Next, we will look for the basis of the eigenvector space $\lambda_4 = -2$. By substituting these values into $det(A(K(1)(2)_3) - \lambda I)$ we get the following:

$$\begin{bmatrix} -\lambda_4 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -\lambda_4 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & -\lambda_4 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -\lambda_4 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & -\lambda_4 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -\lambda_4 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & -\lambda_4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

By eliminating the matrix by the Gauss-Jordan method, obtained:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

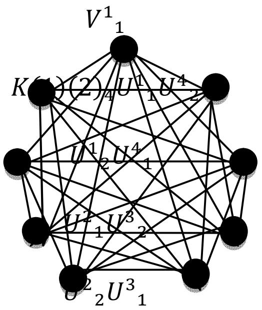
From the results above, that is obvious that the number of bases of the eigenvector space corresponding to $\lambda_4 = -2$ is 2 So that the spectrum of a complete multipartition graph $K(1)(2)_3$ is

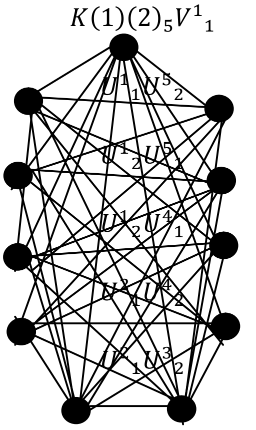
$$Spec\ A(K(1)(2)_3) = \begin{bmatrix} 2+\sqrt{10} & 2-\sqrt{10} & 0 & -2 \\ 1 & 1 & 3 & 2 \end{bmatrix}$$

Spectrum of Complete Multipartition Graphs$\mathbf{K(1)(2)_\alpha}$

In the following table are presented some complete multipartition graph point connectedness matrices $K(1)(2)_\alpha$, with$\alpha \geq 2$

Table 4.2 Characteristic Polynomials of a Complete Multipartition Graph $K(1)(2)_\alpha$

Complete Multipartition Graph $K(1)(2)_\alpha$	**Characteristic polynomials**
	$(1)(\lambda)^4(2+\lambda)^3(-\lambda^2+6\lambda+8)$

	$(1)(\lambda)^5(2+\lambda)^4(-\lambda^2+8\lambda+10)$

Theorem 4.1$K(1)(2)_\alpha$ is the characteristic polynomial of a full multipartition graph.

Proof of the point connectedness matrix of a complete multipartition graph $K(1)(2)_\alpha$ is

$$\mathrm{A}(K(1)(2)_\alpha)=\begin{array}{c|cccccccc} & V^1{}_1 & U^1{}_1 & U^1{}_2 & U^2{}_1 & U^2{}_2 & \cdots & U^\alpha{}_1 & U^\alpha{}_2 \\ \hline V^1{}_1 & 0 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ U^1{}_1 & 1 & 0 & 0 & 1 & 1 & \cdots & 1 & 1 \\ U^1{}_2 & 1 & 0 & 0 & 1 & 1 & \dots & 1 & 1 \\ U^2{}_1 & 1 & 1 & 1 & 0 & 0 & \cdots & 1 & 1 \\ U^2{}_2 & 1 & 1 & 1 & 0 & 0 & \cdots & 1 & 1 \\ \vdots & 1 & 1 & 1 & 1 & 1 & \ddots & 1 & 1 \\ U^\alpha{}_1 & 1 & 1 & 1 & 1 & 1 & \dots & 0 & 0 \\ U^\alpha{}_2 & 1 & 1 & 1 & 1 & 1 & & 0 & 0 \end{array}$$

After obtaining the form of the connected dot matrix, we will look for the eigenvalues and eigenvectors of the matrix using $det(A(K(1)(2)_\alpha)-\lambda I)=0$.And obtained the upper triangular matrix of $(A(K(1)(2)_\alpha)-I\lambda)$ as follows characteristic polynomial of $(A(K(1)(2)_\alpha)-\lambda I)$ is $\det(A(K(1)(2)_\alpha)-I\lambda)$, is the result of multiplying all the main diagonal elements of the upper triangular matrix, so we get:

$$p(\lambda)=(1)(\lambda)^2(2+\lambda)^{\alpha-1}\big((-\lambda)^\alpha+\big((2\alpha)\lambda\big)+(1)(2)\alpha\big)$$

Theorem 4.2 The spectrum of a complete multipartition graph $K(1)(2)_\alpha$ is:

$$\boldsymbol{Spec\,K(1)(2)_\alpha}=\begin{bmatrix}(\boldsymbol{\alpha}-\mathbf{1})+\sqrt{(\boldsymbol{\alpha}^2+\mathbf{1})} & (\boldsymbol{\alpha}-\mathbf{1})-\sqrt{(\boldsymbol{\alpha}^2+\mathbf{1})} & \mathbf{0} & -\mathbf{2} \\ \mathbf{1} & \mathbf{1} & \boldsymbol{\alpha} & \boldsymbol{\alpha}-\mathbf{1}\end{bmatrix}.$$

Evidence From theorem 4.1 obtained that the characteristic polynomials of the complete multipartition graph$K(1)(2)_\alpha$ are:$p(\lambda)=(1)(\lambda)^2(2+\lambda)^{\alpha-1}\big((-\lambda)^\alpha+\big((2\alpha)\lambda\big)+(1)(2)\alpha\big)$

So that eigenvalues are obtained:

$\lambda_1=(\alpha-1)+\sqrt{(\alpha^2+1)},\quad \lambda_2=(\alpha-1)-\sqrt{(\alpha^2+1)},\quad \lambda_3=0\lambda_4=-2.$

Next, the multiplicity of each eigenvalue will be determined. The variety is equal to the space dimension of the eigenvector corresponding to $\lambda_i jika\ i=1,2,3,4$. The size of the eigenvector space corresponding to $\lambda_i, jika\ i=1,2,3,4$, is similar to the number of zero rows in the matrix $(A(K(1)(2)_\alpha)-\lambda I)$ after being reduced to a reduced echelon matrix line.

For $\lambda_1=(\alpha-1)+\sqrt{(\alpha^2+1)}$ matrix $(A(K(1)(2)_\alpha)-\lambda_1 I)$ after being reduced to a reduced row echelon matrix, this results in 1 row zero.

So the multiplicity for $\lambda_1=(\alpha-1)-\sqrt{(\alpha^2+1)}$. Matrix $\big(A(K(1)(2)_\alpha-\lambda_2 I)\big)$. After being reduced to a reduced row echelon matrix, resulting in as many as 1 zero rows. So the multiplicity for $\lambda_2=(\alpha-1)-\sqrt{(\propto^2+1)}$ is 1. For $\lambda_3=0$. matrix$(A(K(1)(2)_\alpha)-\lambda_2 I)$ after row reduction, yields $\propto$ zero rows. So, the diversity for $\lambda_3=0$ is $\propto$,for $\lambda_4=-2$. The matrix $(A(K(1)(2)_\propto)-\lambda_4 I)$ after the reduction to a smaller row Emblem matrix, produces $\propto-1$ zero rows, so the diversity for $\lambda_4=-2$is $\propto-1$. So it is proven that the spectrum of a complete multipartition graph $(K(1)(2)_\propto)$ is obtained:

$$\boldsymbol{Spec\,K(1)(2)_\alpha}=\begin{bmatrix}(\boldsymbol{\alpha}-\mathbf{1})+\sqrt{(\boldsymbol{\alpha}^2+\mathbf{1})} & (\boldsymbol{\alpha}-\mathbf{1})-\sqrt{(\boldsymbol{\alpha}^2+\mathbf{1})} & \mathbf{0} & -\mathbf{2} \\ \mathbf{1} & \mathbf{1} & \boldsymbol{\alpha} & \boldsymbol{\alpha}-\mathbf{1}\end{bmatrix}.$$

Characteristic Polynomial of a Complete Multipartition Graph$\mathbf{K(n)(n+1)_\alpha}$.

The following table presents the characteristic polynomial of the dot-connected matrix of the complete multiplicity graph $K(n)(n+1)_\alpha$ .For$\propto\geq 2$, can be seen in Table 4.3

Table 4.3 Characteristic polynomials of complete multiplicity graphs $K(n)(n+1)_\alpha$.

Matrix	Polynomial Characteristics
$K(1)(2)_\alpha$ $, \propto \geq 2, \propto \in N$	$p(\lambda) = (1)(\lambda)^2(2+\lambda)^{\alpha-1}\big((-\lambda)^2 + \big((2\alpha-2)\lambda\big) + (1)(2)\alpha\big)$

Theorem 4.3 Characteristic polynomials of complete multipartition graphs$K(n)(n+1)_\alpha$is:
$p(\lambda) = (1)(\lambda)^{n\propto+(n-1)}(n+\lambda)^{\alpha-1}\big((-\lambda)^2 + \big(\big((n+1)\lambda\big) + (n)(n+1)\propto\big)\big)$.

Complete Multipartition Graph Spectrum $(\mathbf{n})(\mathbf{n}+\mathbf{1})_{\boldsymbol{\alpha}}$ $\propto = \mathbf{2}, \mathbf{n} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}.$

The following table presents the spectrum of a complete multipartition graph $K(n)(n+1)_\alpha$with $\propto \geq 2$, as shown in Table 4.4

Table 4.4 Spectrum of complete multipartition graphs $K(n)(n+1)_\alpha$

Matrix	Spectrum
$K(2)(3)_\alpha$	$\begin{bmatrix} \left(\frac{3}{2}\propto-\frac{3}{2}\right)+\frac{1}{2}\sqrt{9\propto^2+6\propto+9} & \left(\frac{3}{2}\propto-\frac{3}{2}\right)-\frac{1}{2}\sqrt{9\propto^2+6\propto+9} & 0 & -3 \\ 1 & 1 & 2\propto+1 & \alpha-1 \end{bmatrix}$ $= \begin{bmatrix} \left(\frac{3}{2}\propto-\frac{3}{2}\right)+\frac{1}{2}\sqrt{9\propto^2+6\propto+9} & \left(\frac{3}{2}\propto-\frac{3}{2}\right)-\frac{1}{2}\sqrt{9\propto^2+6\propto+9} & 0 & -3 \\ 1 & 1 & 2\propto-1 & \propto-1 \end{bmatrix}$
$K(3)(4)_\alpha$	$\begin{bmatrix} (2\propto-2)+2\sqrt{\propto^2+\propto+1} & (2\propto-2)-2\sqrt{\propto^2+\propto+1} & 0 & -4 \\ 1 & 1 & 3\propto-1 & \propto-1 \end{bmatrix} =$ $\begin{bmatrix} \left(\frac{3}{2}\propto-\frac{3}{2}\right)+\frac{1}{2}\sqrt{16\propto^2+16\propto+16} & \left(\frac{3}{2}\propto-\frac{3}{2}\right)-\frac{1}{2}\sqrt{16\propto^2+16\propto+16} & 0 & -4 \\ 1 & 1 & 3\propto-1 & \propto-1 \end{bmatrix}$
$K(4)(5)_\alpha$	$\begin{bmatrix} (2\propto-2)+2\sqrt{\propto^2+\propto+1} & (2\propto-2)-2\sqrt{\propto^2+\propto+1} & 0 & -5 \\ 1 & 1 & 4\propto-1 & \propto-1 \end{bmatrix} =$ $\begin{bmatrix} \left(\frac{3}{2}\propto-\frac{3}{2}\right)+\frac{1}{2}\sqrt{24\propto^2+24\propto+24} & \left(\frac{3}{2}\propto-\frac{3}{2}\right)-\frac{1}{2}\sqrt{24\propto^2+24\propto+24} & 0 & -5 \\ 1 & 1 & 4\propto-1 & \propto-1 \end{bmatrix}$

Theorem 4.4 Spectrum of complete multipartition $K(n)(n+1)_\alpha$ is:

$$\begin{bmatrix} \left(\frac{n+1}{2}(\propto-1)\right)+\frac{1}{2}\sqrt{(n-1)^2\propto^2+(2n-2)\propto+(n+1)^2} & \left(\frac{n+1}{2}(\propto-1)\right)-\frac{1}{2}\sqrt{(n-1)^2\propto^2+(2n^2-2)\propto+(n+1)^2} & 0 & -(n+1) \\ 1 & 1 & n\propto-1 & \propto-1 \end{bmatrix}$$

Proof: From theorem 4.4, it is obtained that the polynomial of the complete multipartition graph $K(n)(n+1)_\alpha$ is$p(\lambda) = (1)(\lambda)^{n\alpha+1(n-1)}\left(\big((n+1)+\alpha\big)^{\alpha-1}((-\lambda)^2) + (n+1)\propto - \big((n-1)\propto\big) + (n)(n+1)\propto\right)$

So that eigenvalues are obtained:

$$\lambda_1 = \left(\frac{n+1}{2}(\propto-1)\right)+\frac{1}{2}\sqrt{(n+1)^2\lambda^2+(2n^2-2)\propto+(n+1)^2},$$

$$\lambda_2 = \left(\frac{n+1}{2}(\propto-1)\right)-\frac{1}{2}\sqrt{(n+1)^2\lambda^2+(2n^2-2)\propto+(n+1)^2}, \lambda_3 = 0,\ \lambda_4 = -\big((n-1)\big)$$

Next, the multiplicity of each eigenvalue will be determined. The diversity is equal to the space dimension of the eigenvector corresponding to λ_i, jika $i = 1,2,3,4$. The size of the eigenvector space corresponding to λ_i, jika $i = 1,2,3,4$ equals the number of zero rows in the matrix $(A(K(n)(n+1)_\alpha) - \lambda I)$ after being reduced to an echelon matrix row reduced.

For$\lambda_1 = \left(\frac{n+1}{2}(\propto-1)\right)+\frac{1}{2}\sqrt{(n+1)^2\propto^2+(2n^2-2)\propto+(n+1)^2}$. Matrix$(A(K(n)(n+1)_\alpha) - \lambda_2 I)$ after being reduced to a row-reduced echelon matrix, it yields as many as 1 zero row, So the multiplicity for:

$$\lambda_1 = \left(\frac{n+1}{2}(\propto-1)\right)+\frac{1}{2}\sqrt{(n+1)^2\propto^2+(2n^2-2)\propto+(n+1)^2}. \text{ is 1.}$$

For $\lambda_2 = \left(\frac{n+1}{2}(\propto-1)\right)-\frac{1}{2}\sqrt{(n+1)^2\propto^2+(2n^2-2)\propto+(n+1)^2}$. Matrix$(A(K(n)(n+1)_\alpha) - \lambda_2 I)$ after being reduced to a reduced row echelon matrix, resulting in as many as 1 zero row.

Sothe multiplicity for:$\lambda_2 = \left(\frac{n+1}{2}(\propto-1)\right)-\frac{1}{2}\sqrt{(n+1)^2\propto^2+(2n^2-2)\propto+(n+1)^2}$. is 1. For$\lambda_3 = 0$. The matrix $(A(K(n)(n+1)_\alpha) - \lambda_3 I)$, after being shortened to a shorter row matrix echelon, yields $n\propto(n+1)$ zero rows. So the multiplicity for $\lambda_3 = 0$is $n\propto$

$+(n+1)$, for $\lambda_4 = -(n+1)$. The matrix $(A(K(n)(n+1)_\alpha) - \lambda_4 I)$, after being reduced to a reduced row echelon matrix, yields $(\propto -1)$ zero rows. So the multiplicity $\lambda_4 = -(n+1)$. So it is proved that the spectrum of the complete diversity $(A(K(n)(n+1)_\alpha) - \lambda I)$ is:

$$\begin{bmatrix} \left(\frac{\boldsymbol{n}+1}{2}(\propto -\mathbf{1})\right) + \frac{1}{2}\sqrt{(\boldsymbol{n}-\mathbf{1})^2 \propto^2 + (2\boldsymbol{n}-2) \propto +(\boldsymbol{n}+\mathbf{1})^2} & \left(\frac{\boldsymbol{n}+1}{2}(\propto -\mathbf{1})\right) - \frac{1}{2}\sqrt{(\boldsymbol{n}-\mathbf{1})^2 \propto^2 + (2\boldsymbol{n}^2-2) \propto +(\boldsymbol{n}+\mathbf{1})^2} & \mathbf{0} & -(\boldsymbol{n}+\mathbf{1}) \\ \mathbf{1} & \mathbf{1} & \boldsymbol{n} \propto -\mathbf{1} & \propto -\mathbf{1} \end{bmatrix}.$$

CONCLUSION

The general form of the range of a full multipartition graph $K(n)(n+1)_\alpha$ is in this research; the authors only concentrate on the spectrum on the linked dot matrix, it may be inferred from the findings of the discussion about the adjacency spectrum of a complete multipartition graph. For further research, it is advisable to continue by looking for theorems of various spectrums that can be obtained from the complete graph $K(n)(n+1)_\alpha$ to create a complete graph drawing and find a program.

Acknowledgements

The authors thank to the Postgraduate Program of the University of Brawijaya. This research wasfinancially sup-ported by the Ministry of Education of the Democratic Republic of Timor Leste.

Competing interests

The author has no possible conflicts of interest in this paper's research, authorship, or publishing.

Authors' Contributions

TGF analyzed data, designed research and wrote the manuscript. NH and M designed research, and pre-pared the manuscript.

REFERENCES

Atici M. Computational complexity of geodetic set, Int. J. Comput. Math. 79 (2002),587– 591, doi:10.1080/00207160210954.

Manuel P, Klavžar S, Xavier A, Arokiaraj A and Thomas E. Strong geodetic problems innetworks. DiscussionesMathematicae. Graph. Theory 40 (2020): 307-321 doi:10.7151/dmgt.2139.

Biggs N. 1993. Algebraic Graph Theory. New York: Cambridge University Press.

Biyikoglu T, Leydold J and Stadler PF. 2009. *Laplacian Eigenvectors of Graphs*. Berlin: Springer.

Brouwer AE and Haemers WH. 2012.*Spectra of Graphs Theory and Apllication.* NewYork:

Buhaerah Z. Busrahdan H. Sanjaya. 2022. Teori Graf danAplikasinya.LSQ Makasar.

Bussemaker F, and Cvetkovi´c D. There are exactly 13 connected cubic, integral graphs, Univ. Beograd Publ. Elektrotehn. Fak. Ser. Mat. Fiz. 544–576 (1976) 43–48.

Chartrand G and Lesniak L. 1986.*Graphs and Digraphs Second* Edition.California: aDivision of Wadsworth, Inc.

Chartrand G, Lesniak L, and Zhang P. 2016. Graphs and Digraphs 6th ed. (Florida: CRC Press)

Cvetkovi'c D, Doob M, and Sachs H. Spectra of Graphs Theory and Application, third ed. Johann Ambrosius Barth Verlag, 1995

Glen Marc, Sergey Kitaev and ArtemPyatkin. On The Representation Number of A Crown Graph Discret Applied Mathematics. 2018; 244:95-102.

Harary F and Schwenk AJ. Which graphs have integral spectra? Graphs and Combinatorics, Springer Lecture Notes 406 (1974), 45-51.

Harary F, King A, Mowshowitz and Read RC. Cospectral graphs and digraphs. Bull. London Math. Soc. 3 (1971), 321-328.

John C. Mason and David C. Handscomb. ChebyshevPolinomials. Boca Raton: Chapman & Hall/CRC Press LCC; 2003.

Jones Owen. Spectra of Simple Graphs. Whitman Collonge; 2013.

Kolman B. Elemantary Linear Algebra and Its Applications Ninth Edition. Published by Pearson; 2007.

Noy M and Ribo A. Recursively Constructible Families of Graphs Advances in Appliedathematics.2004; 32:350-363.

****