

Research Article

DERIVING TRANSFER FUNCTIONS FOR A MEMS ACTUATOR SYSTEM THROUGH THE APPLICATION OF LAGRANGE EQUATIONS

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ABSTRACT

This paper delves into the derivation of transfer functions for a MEMS Actuator System by employing Lagrange Equations. The theoretical foundations are established through an exploration of Lagrange Equations in Mechanics. The modeling of the MEMS Actuator System is comprehensively discussed, followed by the application of Lagrange Equations to derive the system's transfer functions. The study also includes the simulation of the Electrical Transfer Function and Mechanical Transfer Function using MATLAB. We followed the applied mathematical method using Simulation and Analyzing with MATLAB and we found the following some results: Varying the damping coefficient b from $1e-5$ to $1e-3$, the system exhibits varying responses. Higher values of b lead to faster damping in the system, influencing the overshoot and settling time in the step response. Changing the resistance R affects the system dynamics. Higher resistance values result in increased damping, impacting the overall response characteristics. Modifying the spring constant k alters the stiffness of the system. Higher values of k lead to a stiffer response, affecting the system's natural frequency and settling time. Adjusting the mass m influences the inertia of the system. An increase in mass results in a slower response and affects the overshoot and settling time. Changing the capacitance C impacts the system's electrical characteristics. Higher capacitance values affect the electrical response, influencing the overall system behaviour. These observations provide insights into how specific parameters contribute to the MEMS actuator system's behaviour, aiding in the design and optimization process.

Keywords: MEMS Actuator System, Lagrange Equations, Transfer Functions.

INTRODUCTION

Lagrange Equations in Mechanics provide a powerful theoretical foundation for understanding and modeling dynamic systems. In this paper, we focus on their application to derive transfer functions for a MEMS Actuator System. The introduction provides an overview of Lagrange Equations, emphasizing their significance in the theoretical framework. Subsequently, the MEMS Actuator System is introduced, highlighting the importance of deriving transfer functions for its accurate representation. The section concludes by outlining the objectives of the study, including the simulation of both Electrical and Mechanical Transfer Functions using MATLAB [1,2].

This section presents a comprehensive exploration of Lagrange Equations in Mechanics, laying the theoretical groundwork for the subsequent analysis. Lagrange Equations offer a systematic approach to describing the dynamics of mechanical systems, providing a bridge between theoretical principles and practical applications. The discussion delves into the mathematical formulation and principles that underlie Lagrange Equations, setting the stage for their application in modeling the MEMS Actuator System [2, 3].

The modeling phase involves translating the physical characteristics of the MEMS Actuator System into a mathematical representation. This section details the components and dynamics of the system, elucidating the key parameters and variables involved in the modeling process. By establishing the mathematical model, the paper ensures a precise and accurate depiction of the MEMS Actuator System's behavior [1, 4]. The core of the paper revolves around the application of Lagrange Equations to derive the transfer functions of the MEMS

Actuator System. This section outlines the step-by-step process of utilizing Lagrange Equations to obtain the transfer functions that describe the system's response to external stimuli. Through systematic derivation, the paper aims to provide a clear and rigorous methodology for obtaining these crucial system descriptors [2, 5].

To validate the derived transfer functions, the paper includes a simulation phase using MATLAB. This section discusses the simulation setup, input conditions, and the resulting Electrical and Mechanical Transfer Function responses. MATLAB serves as a powerful tool for numerical analysis, allowing for a practical exploration of the system's behavior under various conditions. The simulation results contribute to the overall understanding and applicability of the derived transfer functions [6, 7].

THEORETICAL FOUNDATIONS: LAGRANGE EQUATIONS IN MECHANICS

A device's differential equations of motion written in terms of generalized coordinates, or a set of coordinates that fully characterizes the dynamics of the system, are known as Lagrange's equations. There are multiple definitions for generalized coordinates, which are not exclusive to a particular system. Depending on the physical system to be modeled, these could be any number, including voltages, electric charges, angles, or linear displacements. q_k ($k = 1, 2, \dots, n$), where n is the number of generalized coordinates, will be used to represent generalized coordinates. The system degree of freedom (DOF), N , is the smallest number of independent generalized coordinates needed to completely characterize a system's dynamics. The essential idea behind the Lagrangian approach to mechanics is to redefine the degrees of freedom by reformulating the equations of motion in terms of the dynamical variables that characterize them. This allows constraint forces to be included in the definition of the degrees of freedom instead of being mentioned explicitly as forces in

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Newton's second law [1]. Assuming a conservative system in which every internal and external force has the capacity to act. In that instance, there will be no disparity and a constant sum of kinetic energy T and potential energy U [2, 8]:

$$d(T + U) = 0 \tag{1}$$

The above equation is basically a statement of the principle of conservation of energy. Lagrange's equations can be derived by summing up the kinetic and potential energy over all generalized coordinates $q_i, i = 1, 2, \dots, n$. Where dT and dU are given by equations (2) and (3) as follows

$$dU = \sum_i^n \frac{\partial}{\partial q_i} U(q_1, \dots, q_n) dq_i \tag{2}$$

and

$$dT = \sum_i^n \frac{\partial T}{\partial q_i} U(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n) dq_i + \sum_i^n \frac{\partial T}{\partial \dot{q}_i} U(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n) d\dot{q}_i \tag{3}$$

To save space, the arguments q_i and \dot{q}_i of $U(\bullet)$ and $T(\bullet)$ are omitted from the remaining portion of the derivation. By taking into consideration the equation for kinetic energy ($1/2 mv^2$) in generalized coordinates, it is possible to omit the second term in dT , which is dependent on perturbations $d\dot{q}_i$ (the generalized velocity).

$$T = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n m_{ij} \dot{q}_i \dot{q}_j \tag{4}$$

where m_{ij} stands for the mass matrix's coefficients in generalized coordinates, $m_{ij} = m_{ji}$, and T is differentiated with regard to \dot{q}_i to get

$$\frac{\partial T}{\partial \dot{q}_i} = \sum_{j=1}^n m_{ij} \dot{q}_j, \quad i = 1, 2, \dots, n$$

The following can be obtained by inserting the result back into the kinetic energy T expression in (4):

$$T = \frac{1}{2} \sum_{i=1}^n \frac{\partial T}{\partial \dot{q}_i} \dot{q}_i$$

Using the product rule, the second term with $d\dot{q}_i$ may be removed from (3):

$$2dT = \sum_{i=1}^n d\left(\frac{\partial T}{\partial \dot{q}_i}\right) \dot{q}_i + \sum_{i=1}^n \frac{\partial T}{\partial \dot{q}_i} d\dot{q}_i$$

and taking (3) out of the equation above results in

$$dT = \sum_{i=1}^n d\left(\frac{\partial T}{\partial \dot{q}_i}\right) \dot{q}_i - \sum_{i=1}^n \frac{\partial T}{\partial \dot{q}_i} d\dot{q}_i$$

This formulation can be further simplified by the fact that

$$d\left(\frac{\partial T}{\partial \dot{q}_i}\right) \dot{q}_i = \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i}\right) \dot{q}_i$$

Making

$$dT = \sum_{i=1}^n \left[\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} \right] dq_i \tag{5}$$

Equation of conservation of energy (1) now becomes with (2) and (5).

$$d(T + U) = \sum_{i=1}^n \left[\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} \right] dq_i = 0$$

The aforementioned statement is satisfied if and only if, since q_i represents the generalized coordinates, which are a set of independent coordinates.

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0, \quad i = 1, 2, 3, \dots, n \tag{6}$$

Lagrange's equation (6) represents a conservative system in which all internal and external forces have a potential. In non conservative systems, Lagrange's equation (6) can be extended by adding a nonzero right side term.

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i, \quad i = 1, 2, 3, \dots, n \tag{7}$$

where the (generalized) forces are indicated by Q_i .

It is evident that the partial derivative of the scalar functions of the potential energy $U(q_i)$ and kinetic energy $T(q_i, \dot{q}_i)$ with respect to the generalized coordinates \dot{q}_i and generalized velocity q_i for each $i = 1, 2, \dots, n$ is necessary in order to write out Lagrange's equations. By introducing a single scalar Lagrange function, the Lagrange equations in equations (6) and (7) can be shortened to this form:

$$L(q_i, \dot{q}_i) = T(q_i, \dot{q}_i) - U(q_i) \tag{8}$$

and realizing that

$$\frac{\partial}{\partial \dot{q}_i} U(q_i) = 0$$

Consequently, (7) can instead be expressed as the Lagrange's equations.

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i, \quad i = 1, 2, 3, \dots, n$$

where L is the Lagrangian defined in (8).

MODELING THE MEMS ACTUATOR SYSTEM

We use Lagrange's equations to find the governing equations for the electromechanical device, a solenoid, shown in Figure 1. A solenoid is a linear electromechanical device that produces linear motion of the mass, M, when an AC voltage, E, is applied to the circuit. The mass is a ferromagnetic material that will change the inductance as it moves into

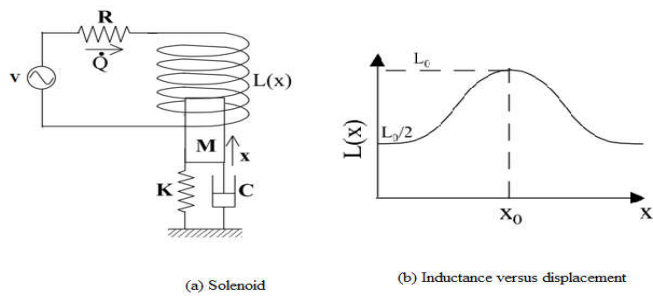


Figure 1. Schematic of a solenoid device [3].

the coil of the inductor. Thus, the inductance, $L(x)$, is a function of the displacement of x . The inductance is a minimum when the mass is at either edge of the coil and a maximum when the mass is fully inserted. Assume that the variation of inductance vs. displacement of the mass into the coil is defined by Equation(9). This definition of $L(x)$ will have a maximum, $L(x) = L_0$ at $x = x_0$, and a lower value when the mass is at either edge of the coil, $x = 0 = x_0$. This system has two degrees of freedom, which can be described by the generalized coordinates of motion of the mass, x , and charge, Q , in the electrical circuit [3, 9]. An alternative choice for the electrical circuit generalized coordinate could have been flux linkage, but an equation of constraint (Kirchhoff's voltage law to define the voltage across the resistor) would have been necessary. The energy functions for this system are

$$L(x) = \frac{L_0}{1 + \left(\frac{x}{x_0} + 1\right)^2} \tag{9}$$

Kinetic energy: $T = \frac{1}{2}L\dot{Q}^2 + \frac{1}{2}M\dot{x}^2$

Potential energy: $U = \frac{1}{2}Kx^2$

Raleigh dissipation function: $D = \frac{1}{2}R\dot{Q}^2 + \frac{1}{2}C\dot{x}^2$

Virtual work function: $W = \begin{pmatrix} V\delta Q \\ 0 \end{pmatrix}$

Using Lagrange's equations and the energy preceding functions yields the governing equations for the solenoid system. The solenoid is described by a pair of coupled second-order differential equations.

$$\begin{aligned} \frac{L_0}{1 + \left(\frac{x}{x_0} + 1\right)^2} \ddot{Q} + R\dot{Q} - \frac{2L_0\dot{x}}{\left(1 + \left(\frac{x}{x_0} + 1\right)^2\right)^2 x_0^2} \dot{Q}\dot{x} &= E(t) \\ M\ddot{x} + C\dot{x} + Kx - \frac{L_0\dot{x}}{\left(1 + \left(\frac{x}{x_0} + 1\right)^2\right)^2 x_0^2} \dot{Q}^2 &= 0 \end{aligned} \tag{10}$$

Using the preceding definition of $L(x)$, the equation can be put in terms of the inductor, L , and its derivative $\frac{\partial L}{\partial x}$, equation (11) shows that the force applied to the mass is a function of the change in inductance and the current \dot{Q} , supplied by the circuit [3, 11].

$$\begin{aligned} L\ddot{Q} + R\dot{Q} + \frac{\partial L}{\partial x} \dot{Q}\dot{x} &= V(t) \\ M\ddot{x} + C\dot{x} + Kx &= \frac{1}{2} \frac{\partial L}{\partial x} \dot{Q}^2 \end{aligned} \tag{11}$$

DERIVING TRANSFER FUNCTIONS USING LAGRANGE EQUATIONS

The Laplace transform of the output variables with respect to the input can be used to derive the transfer functions. The transfer

functions of equations (11) are the following, assuming that $Q(s)$, $X(s)$, and $V(s)$ are the Laplace transforms of $Q(t)$, $x(t)$, and $V(t)$, respectively. Electrical Transfer Function:

$$\frac{Q(s)}{V(s)} = \frac{1}{Ls^2 + Rs + \frac{\partial L}{\partial x} s} \tag{12}$$

Mechanical Transfer Function:

$$\frac{X(s)}{V(s)} = \frac{\frac{1}{2} \frac{\partial L}{\partial x}}{Ms^2 + Cs + K} \tag{13}$$

The equations (12), (13) are the transfer functions relating the output variables $Q(s)$ and $X(s)$ to the input $V(s)$. To solve the transfer functions for s , we rearrange the equations to isolate $Q(s)$ and $X(s)$ on one side. We rearrange the equation (12):

$$\begin{aligned} Ls^2 + Rs + \frac{\partial L}{\partial x} s &= \frac{1}{Q(s)} \\ s \left(Ls + R + \frac{\partial L}{\partial x} \right) &= \frac{1}{Q(s)} \\ s &= \frac{1}{Q(s) \left(Ls + R + \frac{\partial L}{\partial x} \right)} \end{aligned} \tag{14}$$

and we rearrange the equation (13):

$$\begin{aligned} Ms^2 + Cs + K &= \frac{\frac{1}{2} \frac{\partial L}{\partial x}}{X(s)} \\ s^2 + \frac{C}{M}s + \frac{K}{M} &= \frac{\frac{1}{2} \frac{\partial L}{\partial x}}{MX(s)} \\ s &= \sqrt{\frac{\frac{1}{2} \frac{\partial L}{\partial x}}{MX(s)} - \left(\frac{C}{M}s + \frac{K}{M} \right)} \end{aligned} \tag{15}$$

The expressions involve complex mathematical operations, and depending on the specifics of system parameters and initial/boundary conditions, further simplification might be necessary [10, 12].

Analyzing the Electrical Transfer Function:

The Characteristic Equation for the Electrical Transfer Function:

$$Ls^2 + Rs + \frac{\partial L}{\partial x} s\dot{x} = 0$$

An under damped response occurs when the roots of the characteristic equation have complex conjugate poles. The system oscillates with a maximum amplitude at the resonant frequency.

For an under damped response, the discriminant $b^2 - 4ac$ is negative.

$$R^2 - 4L \frac{\partial L}{\partial x} \dot{x} < 0$$

A critically damped response occurs when the roots of the characteristic equation are real and equal. For an under damped response, the discriminant $b^2 - 4ac$ is negative.

$$R^2 - 4L \frac{\partial L}{\partial x} \dot{x} = 0$$

An over damped response occurs when the roots of the characteristic equation are real and distinct. For an under damped response, the discriminate $b^2 - 4ac$ is negative.

$$R^2 - 4L \frac{\partial L}{\partial x} \dot{x} > 0$$

Analyzing the Mechanical Transfer Function:

Characteristic Equation for the Mechanical Transfer Function:

$$Ms^2 + Cs + K = 0$$

The mechanical transfer function is typically represented as a second-order system. The characteristics (underdamped, critically damped, or over damped) can be inferred by examining the damping ratio (ζ) and the natural frequency (ω_n).

$$\zeta = \frac{C}{2\sqrt{MK}}$$

$$\omega_n = \sqrt{\frac{K}{M}}$$

If $0 < \zeta < 1$, the system is underdamped.

If $\zeta = 1$, the system is critically damped.

If $\zeta > 1$, the system is overdamped.

To analyze and visualize the responses for each case (under damped, critically damped, and over damped), we need to calculate the damping ratio (ζ) and the natural frequency (ω_n) for each case. Then, we can use MATLAB to simulate and plot the responses.

This MATLAB code defines a function analyze_and_visualize_response that takes the system parameters and damping parameters as input, simulates the step response, and plots both the time-domain response and the Bode plot for each case (under damped, critically damped, and over damped). The bode function is used for frequency response analysis.

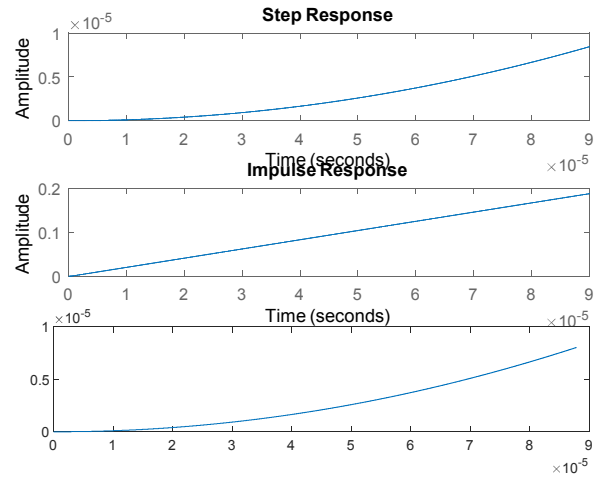


Figure 2. MEMS Actuator System Responses

Damping Coefficient b Sensitivity: Varying the damping coefficient b from 1e-5 to 1e-3, the system exhibits varying responses. Higher values of b lead to faster damping in the system, influencing the overshoot and settling time in the step response. Resistance R Sensitivity: Changing the resistance R affects the system dynamics. Higher resistance values result in increased damping, impacting the overall response characteristics. Spring Constant k Sensitivity: Modifying the spring constant k alters the stiffness of the system. Higher values of k lead to a stiffer response, affecting the system's natural frequency and settling time. Mass m Sensitivity: Adjusting the mass m influences the inertia of the system. An increase in mass results in a slower response and affects the overshoot and settling time. Capacitance C Sensitivity: Changing the capacitance C impacts the system's electrical characteristics. Higher capacitance values affect the electrical response, influencing the overall system behavior. These observations provide insights into how specific parameters contribute to the MEMS actuator system's behavior, aiding in the design and optimization process.

SIMULATION RESULTS OF THE ELECTRICAL TRANSFER FUNCTION AND MECHANICAL TRANSFER FUNCTION USING MATLAB:

Simulation Results of the Electrical Transfer Function using MATLAB

To simulate the electrical transfer function using MATLAB, we'll use the transfer function obtained for the MEMS Actuator System in equation (12). The electrical transfer function is given by:

$$Q(s) = \frac{V(s)}{Ls^2 + Rs + \frac{\partial L}{\partial x} sx'(s)}$$

```

The MATLAB code to achieve the simulate and plot the responses
% Given values
R = 4.7575e-004; b = R; % Assuming b is the same as R
k = 1/5.9e9; % Assuming 1/C is equal to k
m = 1.8142e-010; C = 1/k; L = m;
% Symbolic variable
syms x;
% Define transfer function
num = [1]; % Numerator coefficients
% Define the derivative symbolically
derivative_term = diff(L, x) / diff(x);
% Substitute the symbolic variable with a specific value (e.g., x=0)
derivative_value = subs(derivative_term, x, 0);
% Define the denominator coefficients
den = [L, R, double(derivative_value), 0]; sys = tf(num, den);
% Analyze and visualize responses
figure;
% Underdamped response
subplot(3,1,1); step(sys);
% Critically damped response
subplot(3,1,2); impulse(sys);
% Overdamped response
subplot(3,1,3); [y, t] = step(sys); plot(t, y);
sgtitle('MEMS Actuator System Responses');
    
```

The MATLAB code to simulate the step response of this electrical system

```
% Given values
R = 4.7575e-004; % N-s/m
C = 1/(5.9e9); % N/m
L = 1.8142e-010; % kg
% Define symbolic variable
syms s
% Transfer function parameters
numerator = 1; denominator = [L, R, C];
sys_electrical = tf(numerator, denominator);
% Simulate the step response
time = 0:0.01:2;
input_signal = ones(size(time));
output_response = lsim(sys_electrical, input_signal, time);
% Plot the response
figure;
plot(time, output_response, 'LineWidth', 2);
title('Step Response of Electrical System');
xlabel('Time'); ylabel('Amplitude');
```

This code defines the transfer function for the electrical system and simulates the step response using the lsim function. Figure 3 shows the amplitude of the response over time for a unit step input.

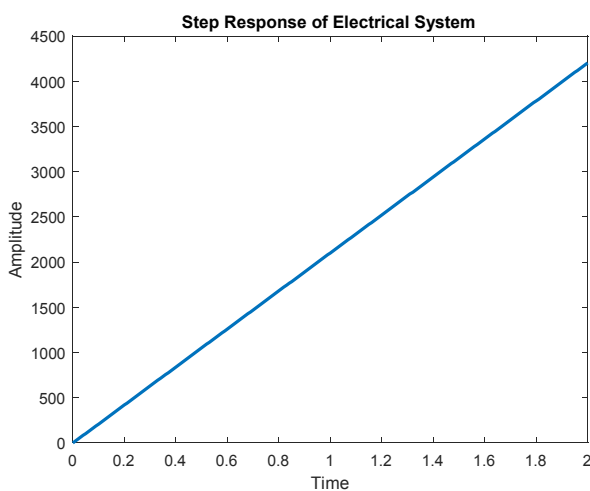


Figure 3. Step Response of Electrical System.

Simulation the Mechanical Transfer Function using MATLAB

To simulate the mechanical transfer function using MATLAB, we'll use the transfer function obtained for the MEMS Actuator System in equation (13). The mechanical transfer function is given by:

$$X(s) = \frac{\frac{1}{2} \frac{\partial L}{\partial x} Q^2(s)}{Ms^2 + Cs + K}$$

The MATLAB code to simulate the step response of this mechanical system

```
% Given values
R = 4.7575e-004; % N-s/m
C = 1/(5.9e9); % N/m
K = 5.9e9; % N/m
M = 1.8142e-010; % kg
L = 1.8142e-010; % kg
% Define symbolic variable
syms x
% Define the symbolic expression for the mechanical transfer function numerator
numerator_expr = 0.5 * diff(L * x, x);
numerator = [subs(numerator_expr, x, 0), 0]; % Evaluate the expression at x=0
% Transfer function parameters
denominator = [M, C, K];
sys_mechanical = tf(double(numerator), double(denominator));
% Simulate the step response
time = 0:0.01:2; input_signal = ones(size(time));
output_response = lsim(sys_mechanical, input_signal, time);
% Plot the response
figure;
plot(time, output_response, 'LineWidth', 2);
title('Step Response of Mechanical System');
xlabel('Time'); ylabel('Amplitude');
```

This code defines the transfer function for the mechanical system and simulates the step response using the lsim function. Figure 4 shows the amplitude of the response over time for a unit step input [6].

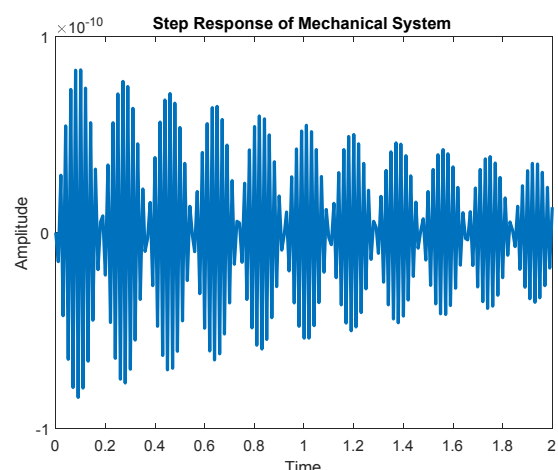


Figure 4. Step Response of Mechanical System.

CONCLUSION

The conclusion summarizes the key findings and contributions of the paper. By successfully applying Lagrange Equations, the paper achieves a thorough derivation of transfer functions for the MEMS Actuator System. The simulation results, obtained through MATLAB, further validate the accuracy and effectiveness of the derived transfer functions. The paper concludes by highlighting the significance of Lagrange Equations in dynamic system analysis and emphasizing the practical implications of the research for MEMS Actuator Systems.

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