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Research Article

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SOLVING TRAVELLING SALESMAN PROBLEM INVOLVING SYMMETRIC TRAPEZOIDAL FUZZY NUMBER USING BRANCH AND BOUND METHOD

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ABSTRACT

In this article, A branch and bound method is proposed for solving fuzzy travelling salesman problem(TSP). In the proposed method costs of travelling are represented by symmetric trapezoidal fuzzy numbers(STFNs). The STFNs defuzzified by applying average ranking function and a new algorithm of proposed method is used to solve fuzzy TSP. To illustrate the proposed method, a numerical example is given and the results are analogized with the results of enduring method.

Keywords: Fuzzy set, Trapezoidal fuzzy number, Symmetric trapezoidal fuzzy number, Average ranking method, Branch and bound method

INTRODUCTION

To overcome the uncertainty in mathematical real life situation, Zadeh [8] was introduced the fuzzy sets in 1965. The TSP is one of the supreme problem in optimization technique. In recent years, fuzzy TSP has got eminent attention and many several technique [2] have been approached to solve TSP. Jadunath Nayak [4] proposed a new method for solving TSP. A different ranking technique was discovered to solve fuzzy TSP [1][3]. Here the average ranking technique is involved for solving fuzzy TSP by applying branch and bound method. The article continue to exists as, Section2 the basic definitions are reviewed. In section3, discuss the mathematical formulation of fuzzy TSP. In section4, a numerical example is given. The paper end with a conclusion in section 5.

BASIC DEFINITIONS

Fuzzy set[5]

A fuzzy set $\hat{A}$ in X is a set of ordered pairs defined by $\hat{A} = \{(x, \alpha_{\hat{A}}(x)) : x \in X, \alpha_{\hat{A}}(x) \in [0,1]\}$, where $\alpha_{\hat{A}}(x)$ is called the membership function.

Trapezoidal fuzzy number [7]

A fuzzy number $\hat{A}$ is a trapezoidal fuzzy number. It's denoted by $\hat{A} = (p, q, r, s)$ with membership function $\alpha_{\hat{A}}(x)$ is given by

$$\alpha_{\hat{A}}(x) = \begin{cases} \dfrac{x-p}{q-p} & , p \le x \le q \\ 1 & , q \le x \le r \\ \dfrac{s-x}{s-r} & , r \le x \le s \\ 0 & , otherwise \end{cases}$$

Symmetric trapezoidal fuzzy number [6]

A fuzzy number $\hat{A} = (p, q, r, r)$ is said to be symmetric trapezoidal fuzzy number if its membership function $\alpha_{\hat{A}}(x)$ is given by

$$\alpha_{\hat{A}}(x) = \begin{cases} \dfrac{x+(r-p)}{r} & , (r-p) \le x \le r \\ 1 & , p < x < q \\ \dfrac{-x+(q+r)}{r} & , q \le x \le (q+r) \\ 0 & , otherwise \end{cases}$$

Ranking technique [4]

If $\hat{A} = (p, q, r, r)$ is a symmetric trapezoidal fuzzy number then the average ranking technique is given by

$$R(\hat{A}) = \frac{p+q+r}{3}$$

For any two trapezoidal fuzzy numbers $\hat{A} = (p, q, r, r)$ and $\hat{B} = (x, y, z, z)$ in $F(R)$, we have the following results

(i) $\hat{A} \ge \hat{B}$ *if and only if* $R(\hat{A}) \ge R(\hat{B})$
(ii) $\hat{A} \le \hat{B}$ *if and only if* $R(\hat{A}) \le R(\hat{B})$
(iii) $\hat{A} \approx \hat{B}$ *if and only if* $R(\hat{A}) \approx R(\hat{B})$

FUZZY TRAVELLING SALESMAN PROBLEM [5]

The fuzzy travelling salesman problem aims to find the order or sequence that the salesman should visit each city, so that the total distance travelled or cost or time of travelling is minimum, with the constraint that the salesman should visit each city once and return to the starting point. The Mathematical Formulation of Fuzzy Travelling Salesman Problem is

$$\text{Minimize } \hat{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \hat{d}_{ij}\, \hat{x}_{ij}$$
Subject to
$$\sum_{j=1}^{n} \hat{x}_{ij} = 1 \ , i = 1,2, \ldots n \ ;$$
$$\sum_{i=1}^{n} \hat{x}_{ij} = 1 \ , j = 1,2, \ldots n$$
$$\hat{x}_{ij} = 0 \text{ or } 1 \text{ for all } i \text{ and } j$$

PROPOSED ALGORITHM

Consider the fuzzy travelling salesman problem with cost represented as symmetric trapezoidal fuzzy number.

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Step 1: Obtain the ranking value of symmetric trapezoidal fuzzy number which is given in the fuzzy TSP.
Step 2 : Reduce the given fuzzy TSP matrix into crisp TSP matrix form using ranking value.
Step3 : Find the lower bound(upper bound) in the reduced matrix for minimization (maximization) problem. Lower(upper) bound = sum of the minimum(maximum) element of each row.
Step 4 : Let branch will occur at X_{AB}.If $X_{AB}=1$, leave row A and column B. and we can't go B→A, so set it to ∞.
Step 5 : Again repeat the step3 and step4 using this reduced matrix, until whole path is found.
Step 6 : Finally we get total distance and shortest path.

NUMERICAL EXAMPLE

Consider a fuzzy travelling salesman problem of four cities with the distance travelled from city i to city j as symmetric trapezoidal fuzzy number

	1	2	3	4
1	∞	(1,2,4,4)	(3,5,8,8)	(7,11,12,12)
2	(1,5,7,7)	∞	(8,12,14,14)	(3,5,6,6)
3	(0,4,7,7)	(5,6,9,9)	∞	(4,12,15,15)
4	(6,8,10,10)	(2,4,5,5)	(9,10,12,12)	∞

Table1. FuzzyTSP

$$\text{Let } \hat{A} = \begin{array}{c} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \end{array} \\ \begin{bmatrix} \infty & (1,2,4,4) & (3,5,8,8) & (7,11,12,12) \\ (1,5,7,7) & \infty & (8,12,14,14) & (3,5,6,6) \\ (0,4,7,7) & (5,6,9,9) & \infty & (4,12,15,15) \\ (6,8,10,10) & (2,4,5,5) & (9,10,12,12) & \infty \end{bmatrix} \end{array}$$

Obtain the ranking value of STFNs by applying average ranking technique, the above matrix is reduced as follows

$$\hat{A} \approx \begin{bmatrix} \infty & 2.3 & 5.3 & 10 \\ 4.3 & \infty & 11.3 & 4.7 \\ 3.7 & 6.7 & \infty & 10.3 \\ 8 & 3.7 & 10.3 & \infty \end{bmatrix}$$

The lower bound of total distance is
LB =2.3+4.3+3.7+3.7=14
Optimal solution will have to be more than or equal to the lower bound of 14.

Iteration 1 :

Now create the branches as $x_{12}=1$, $x_{13}=1$, $x_{14}=1$ and $x_{11}=1$ is a sub tour.
Branch-1 ⟶ For $x_{12}=1$, leave row 1 and column 2
So we can't go 2⟶1, so set it to ∞.

$$\hat{A}_{12} = \begin{array}{c} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{1} & \mathbf{3} & \mathbf{4} \end{array} \\ \begin{bmatrix} \infty & 11.3 & 4.7 \\ 3.7 & \infty & 10.3 \\ 8 & 10.3 & \infty \end{bmatrix} \end{array}$$

Hence, the lower bound of 1 to 2 will be 2.3+4.7+3.7+8=18.7

Branch-2 ⟶ For $x_{13}=1$, leave row 1 and column 3
So we can't go 3⟶1, so set it to ∞.

$$\hat{A}_{13} = \begin{array}{c} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{1} & \mathbf{3} & \mathbf{4} \end{array} \\ \begin{bmatrix} \infty & 11.3 & 4.7 \\ 3.7 & \infty & 10.3 \\ 8 & 10.3 & \infty \end{bmatrix} \end{array}$$

Hence, the lower bound of 1 to 3 will be 5.3+4.3+6.7+3.7=20

Branch-3 ⟶ For $x_{14}=1$, leave row 1 and column 4
So we can't go 4⟶1, so set it to ∞.

$$\hat{A}_{14} = \begin{array}{c} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{1} & \mathbf{3} & \mathbf{4} \end{array} \\ \begin{bmatrix} \infty & 11.3 & 4.7 \\ 3.7 & \infty & 10.3 \\ 8 & 10.3 & \infty \end{bmatrix} \end{array}$$

The lower bound of 1 to 4 will be 10+4.3+3.7+3.7=21.7

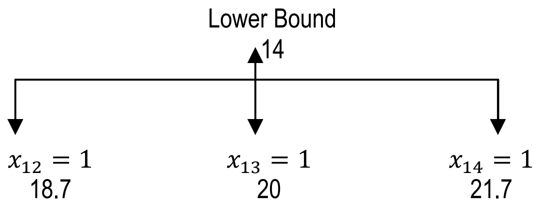

Figure 1. The branch and bound diagram of the Iteration 1

Iteration 2 :

Now create 2 more branches as $x_{23}=1$,$x_{24}=1$and $x_{21}=1$ is a sub tour, continue the above procedure, we get,

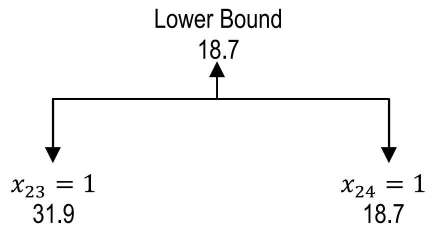

Figure 2 .The branch and bound diagram of the Iteration 2

Iteration 3 :

we create 1 more branches as $x_{31}=1$and $x_{33}=1$ is a sub tour. We have

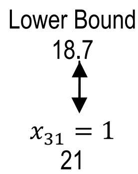

Figure 3. The branch and bound diagram of the Iteration 3

Iteration 4 :

we create 2 more branches as $x_{21}=1$, $x_{24}=1$ and $x_{22}=1$ is a sub tour. We have

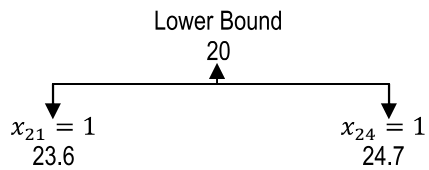

Figure 4. The branch and bound diagram of the Iteration 4

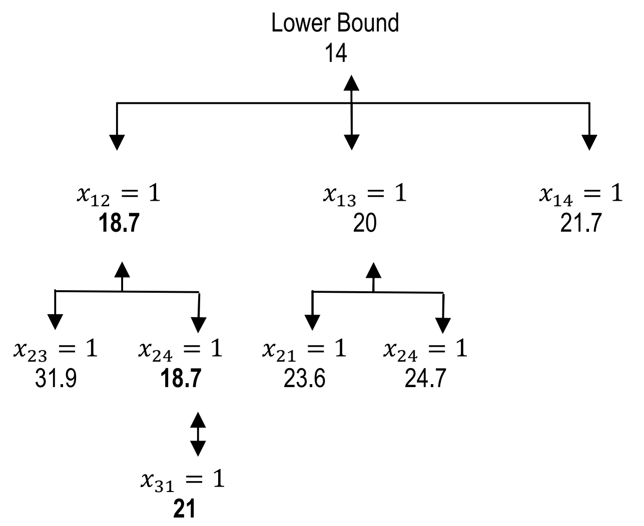

Figure 5. The branch and bound diagram for fuzzy TSP

To complete the travelling city, we create 1 more branches as $x_{43} = 1$

The lower bound of 4 to 3 will be 2.3+4.7+3.7+10.3=21

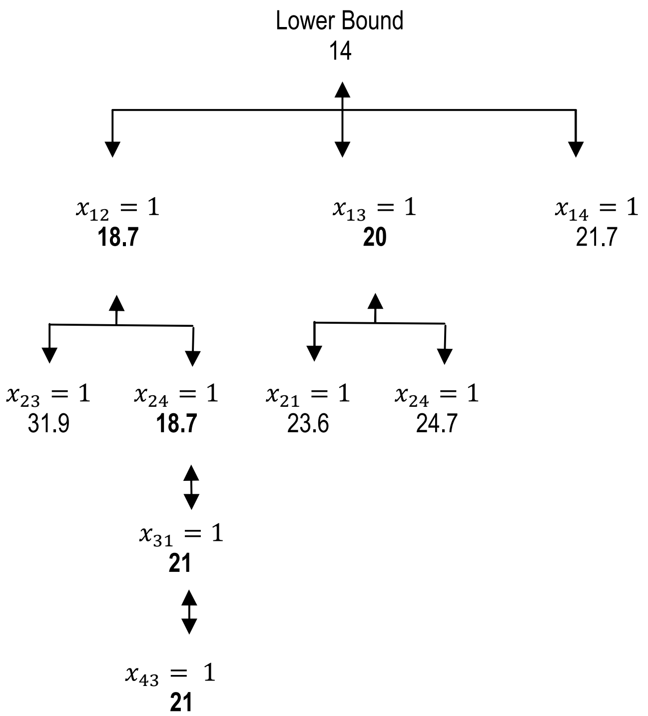

Figure 6. The branch and bound diagram for fuzzy TSP

The best total distance is 1→2,2→4,3→1,4→3
Hence, the shortest path is 1→2→4→3→1
The optimal solution is = $x_{12} + x_{24} + x_{43} + x_{31}$
= 2.3+4.7+10.3+3.7
=**21.**

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Comparative Study :

Table2. Comparative study

S.No.	Methods/Ranking	Yager's Ranking	Proposed Ranking
1	Hungarian	21.2	21
2	Diagonal Completion	24.9	24.7
3	Branch and Bound	21.2	21

CONCLUSION

The travelling cost is consider as symmetrical trapezoidal fuzzy number in this article. The fuzzy TSP has been transformed into crisp TSP. Later, Fuzzy TSP has been solved by branch and bound method. A suitable numerical computation has been discussed and results are compared with existing methods and techniques.

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