

Research Article

BLOCK ADJUSTMENT USING CONTROL DISTANCES CONSTRAINT

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ABSTRACT

Constraints are widely used in photogrammetric studies such as introduction of constraints on interior orientation parameters, exterior orientation parameters and object space coordinates of control points. The value of applied constraint lies in the ability to utilize the information to the greatest extent in reducing the magnitude of error propagation. This paper emphasizes on deriving mathematical models based on using control distances constraint, which implies that each two points in the photogrammetric model should be constrained to a known distance, for simultaneous and self calibration block adjustments. Software's utilizing the derived mathematical models have been developed and tested using mathematical and actual photogrammetric data. The effects of block size, number and location of control distances, camera lens distortion and the random errors on bundle and self calibration block adjustments using the derived mathematical models and the conventional methods have been studied using simulated photogrammetric data. It was found that adding the control distances as constraint improves the accuracy of the adjustment.

Keywords: Aerial photogrammetry, close range photogrammetry, bundle block adjustment, 3D relative coordinate system.

INTRODUCTION

Problems in photogrammetry can be solved mainly by pure mathematical modelling using simple but highly precise coordinates of image points where the instrumentation for measuring these coordinates is conceptually very simple. This method of solution is called analytical photogrammetry. It is universally recognized for having the inherent capability of non-contact and rapid spatial measurements. Broadly speaking, there are two different methods of analytical block adjustment: sequential and simultaneous (bundle). In sequential method, the triangulation is performed in steps analogous to the instrumental method of triangulation [Moffitt and Mikhail, 1980]. The mathematical approaches to the sequential method are, generally, categorized according to the relative or absolute orientation methods employed (e.g. collinearity, coplanarity, scale-restraint conditions) and the method employed for strip or block adjustment (e.g. linear, second- or third-degree polynomial equations, iterative with number of equations, etc.). In the simultaneous method, the block triangulation and adjustment are performed in one step. The desired parameters are adjusted as a result of one simultaneous least squares adjustment of the m photographs (strip or block) by a direct or iterative method. The process is also known as the Bundle Block adjustment or simply Bundle adjustment. The sequential adjustment is advantageous from computational point of view, but its general implication fails to incorporate the full mathematical foundation of a simultaneous adjustment which yields more accurate results [El-Ashmawy, 1999] and also lends itself to statistical assessment with respect to a posteriori precision evaluation and gross errors detection [El-Ashmawy, 1999]. Bundle (block) adjustment may be viewed as the very apex of analytical photogrammetry, by which a variety of problems in the applications of aerial and close range photogrammetry can be solved. Bundle adjustment utilizes the well known collinearity equations, or coplanarity condition [El-Ashmawy, 2021], to establish two equations for each measured image point, and provides a unique solution for the system of observation equations by

the least squares method. The collinearity equations can be written as:

$$\left. \begin{aligned} x_p + F(K)x_p + F(P) \cdot \{P_1(r_p^2 + 2x_p^2) + P_2(2x_p y_p)\} \\ = -f \frac{(X_p - X_o)m_{11} + (Y_p - Y_o)m_{12} + (Z_p - Z_o)m_{13}}{(X_p - X_o)m_{31} + (Y_p - Y_o)m_{32} + (Z_p - Z_o)m_{33}} \\ y_p + F(K)y_p + F(P) \cdot \{P_2(r_p^2 + 2y_p^2) + P_1(2x_p y_p)\} \\ = -f \frac{(X_p - X_o)m_{21} + (Y_p - Y_o)m_{22} + (Z_p - Z_o)m_{23}}{(X_p - X_o)m_{31} + (Y_p - Y_o)m_{32} + (Z_p - Z_o)m_{33}} \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} x_p &= \bar{x}_p - x_o \\ y_p &= \bar{y}_p - y_o \end{aligned} \right\} \quad (2)$$

Where:

\bar{x}_p, \bar{y}_p are the measured photo coordinates of image point p ;

x_o, y_o are the photo coordinates of the principal point;

f is the camera focal length;

X_o, Y_o, Z_o are the object space coordinates of the camera station;

X_p, Y_p, Z_p are the object space coordinates of the object point P ;

m_{11}, \dots, m_{33} are the elements of photo orientation matrix [Ghosh, 2005];

$$r_p^2 = x_p^2 + y_p^2$$

$$F(k) = K_o + K_1 r^3 + K_2 r^5 + K_3 r^7 + \dots$$

= function of symmetrical radial lens distortion;

$$F(P) = 1 + P_3 r_p^2 + P_4 r_p^4 + \dots$$

(4)

= function of symmetrical radial lens distortion; and

Where P_1, P_2 are correction coefficients for decentering lens distortion.

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Equation (1) has three sets of parameters as follows:

- camera interior orientation and lens distortion parameters,
- camera exterior orientation parameters, and
- object space coordinates of points.

Based on the above mentioned parameters, two methods of block adjustment as follows, different in their principles, will arise:

CASE A: when camera interior orientation and lens distortion parameters are known, the block adjustment is called simultaneous, or bundle, block adjustment.

CASE B: when camera interior orientation and lens distortion parameters are not known, the block adjustment is called self calibration block adjustment.

In some cases, however, some known parameters are hidden and may be computationally derivable, whereas in others, some parameters may be known with unequal reliability and it may be advisable to use the known parameters directly or indirectly in the adjustment procedure. The concept of self calibration addresses itself to the former cases, while the utilization of constraints is applicable to the latter cases. Such constraints are meant to enforce the measuring-adjusting process to conform to some functional or geometric relationships or to conform to the degree of reliability as defined by "weighting". The value of such considerations lies in the ability to utilize the information to the greatest extent in reducing the number of unknowns or in reducing the magnitude of error propagations [Mikhail, 1976]. Constraints are widely used in photogrammetric studies such as introduction of constraints on interior orientation parameters [Ghosh, 2005], exterior orientation parameters [Ghosh, 2005], [Wang, 2004] and object space coordinates of control points [Delar *et al.*, 2004], [El-Ashmawy, 2018], [El-Ashmawy, 2021], [El-Ashmawy, 1999], [Ghosh, 2005], [Orun and Natarajan, 1994], [Sarjakoski, 1984]. The available literature review has, only, one application of control distances constraint [El-Ashmawy, 2018] for block adjustment to determine the relative, not absolute, three dimensional (3D) coordinates of points. This application may be suitable for some close range photogrammetric applications but absolutely unsuitable for topographic applications.

Aims of the paper are:

- Derivation of mathematical models based on introduction of constraints on control distances to block adjustment for topographic applications;
- Investigation of the accuracy of the derived mathematical models; and
- Comparing the results of the derived mathematical models and conventional methods for block adjustment.

Derivation Of The Mathematical Models

The developed mathematical models utilize the collinearity equations to establish two equations for each measured image point, and provide unique solution for the system of observation equations by the least squares method.

In Equation (1), the observations are the left and right photo coordinates of an object point. The linearized form of Equation (1), for least squares solution, can be given as follows:

$$V + B \cdot \Delta = \varepsilon \quad (5)$$

where:

- Δ is the correction vector to the current values set for the unknowns (the camera exterior orientation parameters and object space coordinates of the new points for simultaneous block adjustment, or the camera interior orientation parameters, the camera exterior orientation parameters and object space coordinates of the new points for self calibration block adjustment) in the iterative solution;
- B is the matrix of the partial derivatives of Equation (1) with respect to the unknowns;
- V is the residual vector, i.e., the correction vector to the observations; and
- ε is the discrepancy vector.

Introducing constraints to the mathematical models

Constraints are suggested to consider supplemental observation equations [Ghosh, 2005], [Mikhail, 1976] arising from a priori knowledge regarding the object space coordinates of the control points and control distances.

Constraint for control points

Constraint for control points can be written as follows:

$$V^c - \Delta^c = \varepsilon^c \quad (6)$$

where:

- Δ^c is the vector of observational corrections to the object space coordinates of the control points; and
- ε^c is the discrepancy vector, between observed values and current (in iterative solution) values of the object space coordinates of the control points.

Constraint for control distances

Supplemental observation equations for control distances can be derived as described below [El-Ashmawy, 2018].

The distance condition [GHILANI and WOLF, 2017] between two points P and Q can be written as:

$$S_{PQ_m} + V_{S_{PQ}} = \sqrt{(X_Q - X_P)^2 + (Y_Q - Y_P)^2 + (Z_Q - Z_P)^2} \quad (7)$$

Where:

- S_{PQ_m} is the measured distance in object space system between points P and Q;
- $V_{S_{PQ}}$ is the corresponding residual; and
- X_P, \dots, Z_Q are the object space coordinates of points P and Q respectively.

The liberalized form of Equation (7) can be written as:

$$V_S + B_S \cdot \Delta_S = \varepsilon_S \quad (8)$$

In which:

- V_s is the residual vector, i.e., the correction vector to the measured distances;
- Δ_s is the correction vector to the current values set for the unknowns (the object space coordinates of the two ending points of the measured distance) in the iterative solution;
- B_s is the matrix of the partial derivatives of Equation (7) with respect to the unknowns and its elements can be found in [GHILANI and WOLF, 2017]; and
- ϵ_s is the discrepancy vector.

Control distances and their weights can be determined by:

- field measurements followed by computing the weights from observations; or
- computing each distance using the known object space coordinates of the two ending points using Equation (7) and computing its standard deviation, and hence weight, using the theory of error propagation [GHILANI and WOLF, 2017].

The observation equations

Observation equations can be obtained by merging Equations (5), (6) and (8) as:

$$\left. \begin{aligned} V + B \cdot \Delta &= \epsilon \\ V_s + B_s \cdot \Delta &= \epsilon_s \\ V_c - \Delta_c &= \epsilon_c \end{aligned} \right\} \tag{9}$$

Equation (9) can be rewritten as:

$$\bar{V} + \bar{B} \cdot \Delta = \bar{\epsilon} \tag{10}$$

In which

$$\bar{V} = \begin{bmatrix} V \\ V_s \\ V_c \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ B_s \\ -I \end{bmatrix} \quad \text{and}$$

$$\bar{\epsilon} = \begin{bmatrix} \epsilon \\ \epsilon_s \\ \epsilon_c \end{bmatrix}$$

The principle of the least squares method requires the minimizing of the quadratic form $V^t \cdot W \cdot V$, where W is the weight matrix whose elements are the weights associated with each of the observations. The least squares solution of an equation similar to Equation (10) to compute the values of Δ and the necessary statistical data is available in [El-Ashmawy, 1999].

Developing The Necessary Software

The current research includes the development of two software's and their main tasks are tabulated in Table 1.

Table 1. The Developed Software's & Their Main Tasks

SOFTWARE	MAIN TASK
Col_Dis_Con_Consts	Simultaneous block adjustment using distances and control points constraints
Col_Cal_Dis_Con_Consts	Self calibration block adjustment using distances and control points constraints

These software's provide an access to major computational phases of analytical block triangulation. The main functions of the developed software's are:

- Data preparation: It performs the necessary tasks for preparing the data to start block adjustment [El-Ashmawy, 1999].
- Iterative least squares solution for performing the specified task as shown in Table 1. This includes the computations of the adjusted values of unknowns, depending on the specified task, residuals of photo and object space coordinates of control points, if any, and variance of unit weight.
- Computation of statistical data: It includes the computation of the necessary data for statistical analysis and error detection [El-Ashmawy, 1999] such as variance of unit weight, cofactor and covariance matrices for unknowns, depending on the specified task, adjusted photo coordinates and their cofactor matrix, residuals of photo coordinates, dimensions of error ellipses, etc.

The software's utilize efficient techniques of Data Structuring [Malik, 2010], Random File Access and Dynamic Memory Allocations [Gregory, 1998] for automatic processing and representation of the data and results. The software's are window-driven type for facilitating its execution to the user [Gregory, 1998].

Software's For Testing The Results Of The Derived Mathematical Models

For comparing the accuracy of the results of the derived mathematical models and the conventional methods, the following software's, Table 2, are used in this research.

Table 2. Software's for comparing the results of the derived mathematical models

SOFTWARE	MAIN TASK
PHOTOMAP[El-Ashmawy, 1999]	Simultaneous block adjustment with control points constraints
Col_Cal_Consts [El-Ashmawy, 2021]	Self calibration block adjustment with control points constraints

EFFECTS OF ERRORS ON THE ACCURACY OF ADJUSTMENT

The mathematical photogrammetric data can be advantageously used for testing of photogrammetric methodologies and systems since in this case error free input data and end results are both known [El-Ashmawy, 2018],[El-Ashmawy, 2018],[El-Ashmawy, 2021],[El-Ashmawy, 1999]. MATHP software [El-Ashmawy, 1999] has been used for generating the necessary mathematical blocks of photographs.

The present work concentrates on studying the effect of the random and lens distortion errors on the results of block adjustment.

Effect of the random errors

The effect of the random errors was tested by numerical simulation as following:

- Generating error free photogrammetric data of blocks of different sizes using MATHP software.
- Generating normally distributed error(s) with arbitrary mean(s) and standard deviation(s) as presented in [El-Ashmawy, 2021]. The obtained errors were then applied to the error free

- photo coordinates and ground coordinates of control points of the generated blocks.
- The distances and their standard deviations were computed using Equations (7) and (9) as explained earlier.

- Finally, simultaneous block adjustments with/without adding control distances constraint were performed to adjust the available blocks and the results, in the form of standard deviation of unit weight ($\hat{\sigma}_o$), Root Mean Square Error (RMSE) and Maximum Absolute Error (MAE) values at all distances and ground coordinates of points, were obtained and tabulated in Table 4.

Table 4. Results of Simultaneous block adjustment (case of random errors only)

Constraints	Block Title	Distances*			Ground Coordinates of Points*					
		$\hat{\sigma}_o$	RMSE	MAE	X		Y		Z	
					RMSE	MAE	RMSE	MAE	RMSE	MAE
Control Distance & Control Points	Model	0.97	5.900	20.612	3.648	08.520	4.392	09.842	6.649	13.420
	1Strip	1.09	5.700	19.181	4.184	10.971	4.210	10.397	6.185	16.485
	2Strip	1.05	4.000	15.735	2.593	07.032	3.087	08.870	5.028	18.948
	3Strip	1.12	4.100	22.082	2.420	06.787	2.992	16.091	5.051	18.371
	4Strip	1.09	3.800	19.190	2.361	08.306	2.565	11.852	4.040	15.214
	5Strip	1.07	3.700	17.141	2.264	07.710	2.328	11.455	4.687	18.856
Control Points	Model	0.93	6.900	24.396	3.824	08.906	4.544	10.512	6.770	14.013
	1Strip	1.04	6.200	22.408	4.213	12.169	4.228	10.493	6.383	17.320
	2Strip	1.00	4.400	17.682	2.655	07.158	3.224	09.723	5.118	19.696
	3Strip	1.03	4.600	25.761	2.216	06.827	3.097	17.263	5.212	18.893
	4Strip	1.01	4.300	23.196	2.367	09.202	2.701	12.723	4.266	17.172
	5Strip	0.98	4.200	19.026	2.287	07.786	2.775	11.634	4.767	21.534

* Values in μm at Photo Scale 1:1

From Table 4, the following conclusions can be obtained:

- The derived mathematical models are suitable for simultaneous block adjustment for a block of photographs of any size.
- There is no significant difference between the a posteriori standard deviation ($\hat{\sigma}_o$) and the a priori standard deviation ($\sigma_0 = 1.0$) and hence that the correct simulation assumptions and block adjustment have been achieved.
- Adding control distances constraints has significant effect on improving the accuracy and reducing the MAE values of the obtained results.

Effect of the lens distortion errors

As has been mentioned, lens distortion consists of two components: symmetric lens distortion (Equation (3)) and asymmetric lens distortion (Equation (4)). The lens distortion errors were introduced to the blocks of mathematical photographs as follows:

- Generating error free photogrammetric data of blocks of different sizes using MATHP software.
- Assigning values for the lens distortion coefficients and generating errors in the range of 50 μm using Equations (3) and (4).
- Adding the generated errors to the error free photo coordinates
- Computing the distances and their standard deviations as explained earlier.

The results of self calibration block adjustments are shown in Table 5.

Table 5. Results of self calibration block adjustment (case of lens distortion errors)

Constraints	Block Title	Distances*		Ground Coordinates of Points*					
		RMSE	MAE	X		Y		Z	
				RMSE	MAE	RMSE	MAE	RMSE	MAE
Control Distance & Control Points	Model	0.100	0.170	0.035	0.083	0.042	0.129	0.069	0.162
	1Strip	0.000	0.098	0.023	0.070	0.023	0.052	0.039	0.104
	2Strip	0.000	0.116	0.021	0.068	0.021	0.057	0.038	0.117
	3Strip	0.000	0.107	0.018	0.067	0.020	0.064	0.036	0.105
	4Strip	0.000	0.110	0.017	0.066	0.018	0.070	0.036	0.111
	5Strip	0.000	0.108	0.018	0.067	0.018	0.065	0.036	0.119
Control Points	Model	0.100	0.182	0.037	0.089	0.044	0.137	0.071	0.167
	1Strip	0.000	0.101	0.025	0.072	0.025	0.055	0.042	0.114
	2Strip	0.000	0.123	0.022	0.068	0.023	0.060	0.040	0.120
	3Strip	0.000	0.117	0.020	0.068	0.022	0.075	0.038	0.117
	4Strip	0.000	0.113	0.019	0.066	0.020	0.071	0.038	0.117
	5Strip	0.000	0.111	0.020	0.067	0.019	0.067	0.038	0.127

* Values in μm at Photo Scale 1:1

In this case, error free photogrammetric data of blocks of different sizes using MATHP software were generated, and random and lens distortion errors were generated and applied to the error free photo coordinates and ground coordinates of control points of the generated blocks as explained earlier.

Table 6 illustrates the results of self calibration block adjustments, for this case.

From Tables 5 and 6 the following conclusions can be drawn:

- The derived mathematical models are suitable for self calibration block adjustment for a block of photographs of any size.

- Lens distortion errors have significant effect on the accuracy of block adjustment especially for Z-coordinates determination. Control distances constraint compensates the lens distortion errors slightly better than without using it. This opens the door to use this type of constraint for camera calibration methods.
- Adding control distances constraint improves the accuracy of the obtained results. It has significant effect on reducing the values of MAE especially for the distances and Z-coordinates determinations.

Table 6. Results of self calibration block adjustment (case of random and lens distortion errors)

Constraints	Block Title	$\hat{\sigma}_o$	Distances*		Ground Coordinates of Points*					
			RMSE	MAE	X		Y		Z	
					RMSE	MAE	RMSE	MAE	RMSE	MAE
Control Distance & Control Points	Model	1.02	4.700	14.777	3.225	08.122	3.357	06.792	6.785	16.810
	1Strip	1.05	6.300	22.973	4.560	13.067	4.353	12.326	6.913	19.469
	2Strip	1.06	4.000	16.095	2.632	07.660	3.135	09.391	5.470	21.898
	3Strip	1.12	4.100	22.087	2.223	06.534	2.994	16.158	5.169	17.684
	4Strip	1.09	3.600	19.097	2.415	07.940	2.569	12.087	4.138	14.907
	5Strip	1.07	3.600	16.479	2.258	06.812	2.333	11.631	5.045	18.285
Control Points	Model	0.98	5.100	16.096	3.458	08.802	3.527	07.643	7.466	18.270
	1Strip	1.00	6.500	24.719	4.697	15.075	4.510	13.364	7.251	21.834
	2Strip	1.00	4.000	17.962	2.529	07.862	3.282	09.998	5.725	23.546
	3Strip	1.04	4.200	23.155	2.197	06.324	3.110	17.608	5.354	18.756
	4Strip	1.01	3.800	21.064	2.419	08.991	2.680	13.118	4.425	17.088
	5Strip	0.98	3.800	17.670	2.275	06.766	2.466	11.817	5.133	19.814

*Values in μm at Photo Scale 1:1

Testing The Derived Mathematical Model For Control Points Extension

The derived mathematical model has been tested for control points extension. The used actual photogrammetric data [El-Ashmawy, 1999] consists of a pair of stereo photographs taken by Wild Avioplot RC10 Automatic Camera System of Echallens of wide angle coverage on a 23 × 23 cm format at 620m height with focal length 153.18mm lens. The average photo scale is about 1:4300. The camera calibration data e.g. calibrated focal lens, calibrated fiducial marks and radial lens distortion are available [El-Ashmawy, 1999]. The measurement of image points coordinates was carried out [El-Ashmawy, 1999] on the stereo comparator of Aviolyt BC2, Leica, Switzerland, having a least count of 1 μm . The area contains 16 well-distributed and identified control points. The control point numbers, ground coordinates and standard errors are also available. Studying the accuracy of the control extension was performed by using three different patterns of control points [El-Ashmawy, 1999]. The objectives of using different control point patterns were:

- Determination of the effect of control points number and location on the accuracy of the generated control points (check points), and
- Comparison between the results of simultaneous and self calibration block adjustments.

The block adjustment was performed as follows:

- Simultaneous block adjustment using control points, and control points and distances as constraints. For simultaneous block adjustment, the available camera calibration data was introduced to the adjustment.
- Self calibration block adjustment using control points, and control points and distances as constraints.
- Tabulating the final results in the form of RMSE and MAE values for all distances and ground coordinates as depicted in Table 7.

From Table 7, the following conclusion can be drawn:

- Increasing the number of control points improves the obtained accuracy;
- Self calibration block adjustment method improves, for the used data, the results of block adjustment; and
- The results of the proposed method are comparable or better than the results of the conventional methods which use only control points as constraint.

Table 7. Results of block adjustment using actual data

Block Adjustment Method	Constraints	Control Points	Control Distances	Distances (in cm)		Ground Coordinates of Points (in cm)					
				RMSE	MAE	X		Y		Z	
						RMSE	MAE	RMSE	MAE	RMSE	MAE
Simultaneous	Control points	6	--	9.96	23.43	6.99	15.51	5.55	09.70	11.01	27.41
		9	--	9.63	25.03	6.09	14.42	4.75	06.96	10.32	26.63
		12	--	9.69	25.84	5.39	12.96	4.35	06.01	09.72	17.56
	Control points & Distances	6	15	8.72	21.97	6.52	14.45	5.31	09.26	10.30	25.53
		9	36	8.98	22.37	5.78	13.68	4.49	06.41	10.24	23.91
		12	66	8.64	24.36	5.29	12.56	4.10	05.54	09.19	16.46
Self Calibration	Control points	6	--	8.68	20.50	5.82	12.72	4.85	10.19	9.35	21.34
		9	--	8.60	21.45	5.13	11.42	4.45	12.35	9.09	16.87
		12	--	8.57	22.16	5.23	12.16	4.17	13.50	8.48	18.56
	Control points & Distances	6	15	8.13	19.01	5.42	11.95	4.53	09.56	8.89	20.39
		9	36	7.95	19.58	4.71	10.30	4.10	11.50	8.63	16.11
		12	66	7.81	19.93	4.72	10.54	4.05	12.52	8.01	17.51

Conclusions & Recommendations

- Using control distances constraint is applicable to simultaneous and self calibration block adjustment for blocks of photographs of any size.
- Adding control distances constraint compensates the lens distortion errors slightly better than without using it.
- Using control distances constraint slightly improves the accuracy of the results of simultaneous and self calibration block adjustment.
- This paper shows the necessity for the mathematical photogrammetric data for testing the photogrammetric methods and software's

It is recommended to study the effect control distances constraint on camera calibration techniques.

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