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Research Article

THE INTRODUCTION OF DIGITAL RESOURCES IN HIGHER INSTITUTIONS OF TECHNOLOGICAL STUDIES: THE RESOLUTION EXAMPLE OF DIFFERENTIAL EQUATIONS IN MAPLE ENVIRONMENT

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ABSTRACT

This paper presents a didactic analysis of instrumented practices of 1styear university students around the resolution of differential equations in the Maple environment. This analysis attempts to articulate the anthropological theory of didactics and the instrumental approach to highlight the difficulties that may be encountered in the transposition of a mathematical resolution to a computer solution of a problem. We use on the theoretical and experimental levels the model of the double transposition of Broner and Briant (2015), in the case of the Euler method. Our methodology is carried out within the framework of a "mathematics workshop" designed by a higher institution of technological studies in Tunisia, through an analysis of written and digitized productions of student pairs in response to a control duty proposed by the teacher.

Keywords: instrumented practices, differential equations, computer environment, transposition.

INTRODUCTION

This research contributes to a reflection on the impact of educational t echnologies in higher education and the use of digital resources by st udents.The current reform in Tunisia encourages the integration of IC Ts (Information and Communication Technologies) into the practice of university institutions in the hope of improving the quality of student training. This study is part of a research in mathematics didactics conducted at a higher institution of technological studies in Tunisia which provides vocational education. This institution integrates a component of mathematics teaching units entitled:"Applie d Mathematics Workshops" for first- year students of Electrical Engineering and Mechanical Engineering. These workshops are organized in hybrid mode, i.e. qt the interface of mathematics and computer science and in the classical (pencil / paper) and computer environments. They are provided by mathematics teachers and aim to reinforce certain concepts encountered in classical mathematics courses by means of symbolic calculus software imposed by the institution such as Maple or Matlab. In the "subject sheets" intended for teachers we read:

> "*This workshop aims to develop the learner's ability to solve concrete problems, i.e. to read a statement, analyze it, understand it, mathematically transcribe it, find the solution and interpret it using MAPLE or MATLAB*".

(Fact Sheet, EG Licensing Evaluation Commission Report, 2019). In this context, we were interested in the potential effects of ICT integration on the instrumented practices of students around a particular study theme, that of solving first - order differential equations (DE) using the Euler's method in the Maple computer environment. Indeed, this teaching theme is suggested by the mathematics programs of the first year and has the advantage of being both interdisciplinary (mathematics-physics and computer science) and unifier of several mathematical notions such as the notions of variable, function, and graph, primitive... We place at the heart of our problem the reasons which would explain some recurrent errors made by students when they move to a computer transposition of the problem posed, and which may be related to difficulties in conceptualization of the mathematical concepts involved which would influence the development of an algori thm of resolution adequate to the instrumental action. In this way, does the use of ICT in general help to shed light on certain erroneous conceptions of mathematical notions that the classical environment (pencil/paper) does not always allow to identify? In this article, we present the results of an analysis of the instrumented practices of students engaged in pairs in solving a mathematical problem proposed by their teacher for evaluation. We highlight the approaches implemented by the students to move first from a mathematical resolution of the problem to an algorithmic resolution, then to a computer resolution via the application on machine. Is the use of digital tools by students reasonable? Does it validate or invalidate resolution techniques developed in the pencil-paper environment? How to interpret the difficulties encountered at each stage of the process of the dual didactic and computer transposition? We begin by presenting the theoretical approaches that underpinned our research and then the didactic and epistemological issues relating to DE resolution via the Euler method in both environments (classical and computer). We then present our analysis methodology, the experimentation we conducted and the main results relating to student practices on the basis of an analysis of written traces and digitized files collected.

THEORETICAL FRAMEWORK: AN ARTICULATION OF TWO APPROACHES

This research is part of the didactics of mathematics and is based on both the anthropological theory of didactics (ATD, Chevallard, 1999) and the instrumental approach (Rabardel, 1995). The ATD makes it possible to model any mathematical activity in terms of praxeologies

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which are available in mathematical organizations (MO) and didactic organizations (DO) of the knowledge to be taught. In this perspective, any activity is made up of tasks referring to types of tasks which are accomplished by means of techniques. These are justified by technologies, which are themselves legitimized by a theory. For example, the T-task: “For example the task of type T: "design a program to solve a mathematical problem” requires a technique which is based on the performance of two types of tasks: T1: "to create a computer algorithm" or "to adapt a mathematical algorithm" from the mathematical resolution of the problem and T2: "to adapt the computerized algorithm into a program computer, installed on a given computer system. The underlying technology refers to the process of computer programming justified by computer theories. Moreover, to answer the question of what is formed a given technique? What ingredients is it made of? The ATD introduces the concepts of ostensive and non-ostensive in a mathematical knowledge modeling problem. Mathematical activity is considered a mental and intellectual activity because it solicits reason; reasoning, ideas, and intuitions with few materials which is considered as an aid to activity such as writing, graphics, words, speech ... have their specificity as signs occupying the place of other objects that they represent. Thus, mathematical activity is conditioned by material, visual, sound and tactile instruments that it brings into play and which refer to ostensive. This dimension, which is indispensable for the construction of a concept, is considered the instrumental function in the construction of a notion and refers to non-ostensive. Ideas, intuitions, notions, concepts are non-ostensive that can be can be solicited and evoked by manipulating the ostensive associated with them.. In the analysis of mathematical activity, the ostensive / non-ostensive dialectic is often conceived in terms of signs and meaning: ostensive objects are signs of non-ostensive objects which constitute their meaning or significance. Considering a technique as a regulated manipulation of ostensive objects, Rabardel (1995) considers them as "artifacts" of a material or symbolic nature, that their manipulation makes them “instruments”. Therefore an ostensive object can be considered as a possible instrument of human activity that is to say as an entity which makes it possible, in association with others, to conform techniques allowing certain tasks to be accomplished. The distinction between “artifact” and instrument highlighted by the instrumental approach (Rabardel, 1995) allows two crossed processes to be modeled and intertwined in a mathematical activity: instrumentalization and instrumentation. The first relates to the personalization of the artifact by the subject (user) and the second relates to the emergence of subject’s *schemes* of use (i.e. the way in which the artifact will contribute to pre-structure the user's action, in order to carry out the task in question). Béguin and Rabardel (2001) distinguish three types of utilization schemes: use schemes referring to the subject's interaction with the artifact, instrumented action schemes directed towards the object of the activity and summoning the use schemes to achieve the goals pursued and schemes of instrumented collective action, referring to the use of artifacts by several subjects, simultaneously or jointly. In our study, these two theoretical approaches are articulated by means of the concepts of ostensive and non-ostensive with the objective of pointing out the difficulties encountered by the students to implement a mathematical organization around an object of knowledge in the computer environment compared to those developed in the classic pencil-paper environment. In this perspective, the consideration of the algorithmic aspect is necessary to better approach the possible confusions between the ostensive in the resolution of the mathematical problem with a view to its computer transposition. Briant and Bronner (2015), highlight the distance that separates the initial activity of solving a mathematical problem in the usual environment (pencil-paper), from its programming and make a further distinction between a computerized algorithm (the "pseudo-code" language) and a "computer program" (in computer language). Indeed, “when a task such as 'designing a program to solve a problem' is given, we see the emergence of a double didactic and computer transposition associated with different techniques, justified by technologies relating to the mathematical domain, the computer domain, or both together. "(Briant, Bronner, 2015, p236). Three types of resolution are then defined during this process, the mathematical resolution engaged in the classical pencil-paper environment which will give rise to a first algorithm (mathematical algorithm), the algorithmic resolution which consists in transposing the mathematical algorithm into a computer algorithm, written in pseudo-code (first transposition) and computer resolution, which consists in transposing the computer algorithm into a program via a computer language adapted to the software (second transposition). We hypothesize that the techniques implemented by the students to solve ED by the Euler method, in the classical and computer environments suggest difficulties of conceptualization of certain mathematical notions related to the numerical resolution that the classical environment alone would not have made it possible to identify.

AN EPISTEMOLOGICAL ANALYSIS OF THE ISSUES R ELATED TO THE APPLICATION OF THE EULER’S M ETHOD

description of the technique

Euler’s method is an opportunity to develop numerical resolution skills among students. The first order differential equation satisfying the conditions of existence and uniqueness of a solution is returned to the form: (E): y'=f(x,y) where f is a numeric function with two realva riables .The solution when it exists on an interval I represents a functi on u that can be derived on I and verifying: «∀x∈I,u'(x)= f(x,u(x))».

the implementation of such a technique highlights an approximate solution which takes the form of a piecewise affine function on I. Thus, the student manipulates ostensive which refer to functions of different kinds: the numerical function y of the variable x, the numerical function f with two real variables (x, y), the piecewise affine function (Euler's solution) and the exact solutions function in the case of an DE, accessible with algebraic techniques. It is a procedure for approaching the solution of a first order LDE with an initial condition presented as:

$$\begin{cases} y' = f(x,y), x_0 \le x \le x_0 + T \\ y(x_0) = y_0 \end{cases} \quad \text{(Cauchy-problem).}$$

The mathematical objects and the relations involved are generally accessible for first year university students, moreover this method has an algorithmic and programmable character which in some way justifies the raison d'être of this approach in the programs of these institutions. academics. Like any other numerical method, it is an approximate method based on the discretization of the variable x involved , which justifies in some way the reason of being for this approach in the programs of these university institutions. as any other numerical method, it is an approximate method based on the discretization of the variable x. The interval $[x_0, x_0 + T]$ is divided into a number N of subdivisions of the same length, $h = \frac{T}{N}$, finally the Euler’s method returns a list $(y_0, y_1, \ldots, y_N)$ of approximate values of $y(x_i)$, whereas $x_i = x_0 + ih$, $i \in [\![0, N]\!]$and y designates the "exact" solution of the DE in question The polygonal curve (Euler’s curve) connecting the points $M_i(x_i, y_i)$, is a graphical approximation of “exact solution” curve (curve of the function y), it offers an approximate view of the behavior of such a solution over the interval $[x_0, x_0 + T]$. Thus, if it is desired that this vision be good, the choice of the discretization step h must be sufficiently small, and consequently the number N of iterations (the number of values to calculate) can become large, which requires a certain economy and speed of calculation and a considerable memorization, hence the need for the computer tool. Euler's method assumes that the intended solution y of the Cauchy-problem exists and for a reasonably small value of h, and $x \in [x_0, x_0 + T[$, the expression "y(x)+h.f(x,y(x)" is a good approximation of y(x+h) i.e. $y(x+h) \approx y(x) + h.f(x, y(x))$ (Euler's formula). This formula can be obtained by the fact that y the solution aimed at is derivable over the interval $I = [x_{0,}x_0 + T[$, consequently it allows limited development at least to order 1 in the vicinity of any point x of this interval, which allows writing: $y(x+h) = y(x) + h.y'(x) + o(h)$, or simply by translating the derivability of this solution at any point $x \in [x_0, x_0 + T[$ to the expression : $\lim_{h \to 0} \left(\frac{y(x+h) - y(x)}{h}\right) = y'(x)$, then for $h \approx 0$, $\frac{y(x+h) - y(x)}{h} \approx y'(x) = f(x, y)$ or by integrating the differential equation over the interval $[x, x+h]$, for h small enough:

$$y(x+h) - y(x) = \int_x^{x+h} y'(s)ds = \int_x^{x+h} f(s, y(s))ds \approx h.f(x, y)$$, as shown in the following graph

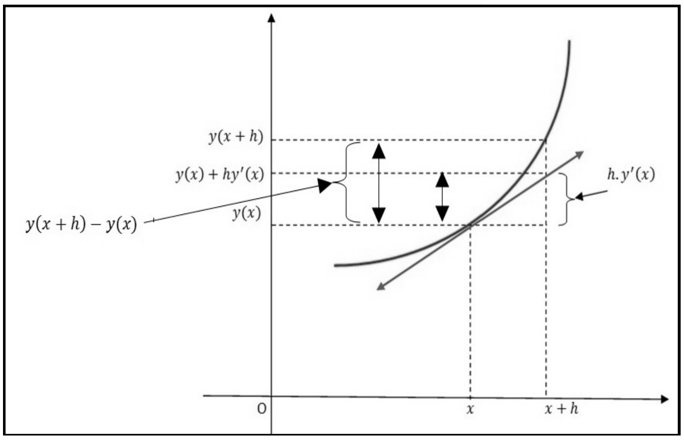


Figure 1. Graphical illustration of Euler's method

Epistemological issues related to the application of the Euler me thod in paper-pencil and digital environments

The implementation of the Euler method involves three stages: the first consists in constructing a series of points from the differential equation, it is a transition from the continuous to the discrete, the sec ond consists in determining a list or a table of numerical values and th e last consists in the graphical construction and thus refers to a transit ion from the discrete to the continuous. Each stage is based on a change in semiotic representation registers (Duval, 1996). For Duval, the only way to access mathematical objects is through semiotic representations.

> "Semiotic representations are productions consisting of the use of signs belonging to a (semiotic) system of representatio n which has its own constraints of meaning and functioning"

(Duval, 1993). We identify the algebraic, numerical and graphical registers for the resolution of a DE by the Euler method. The transition from the differential equation to the numerical sequence is is carried out in the in the pencil-paper environment by applying the sequence of Euler's terms, and the other two stages are carried out in the computer environment. We summarize the characteristics of numerical resolution by Euler's method according to the teaching environment in the following table.

Pencil-paper environment		Maple- environment	
Stage1		Stage2	Stage3
continuous algebraic register →	Discrete algebraic register →	Numerical register →	Graphic register
differential equation $\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$	numerical sequences $x_{k+1} = x_k + h$ $y_{k+1} = y_k + h.f(x_k, y_k)$	list of approximate n umeric values of the x_k and they_k	discrete isolated point graph of the points $M_{k(x_k, y_k)}$ ↓ Euler's curve, i.e. Polygonal curve connecting the points$M_{k(x_k, y_k)}$

Table1. Registers of Semiotic Representations Involved in the Euler's M ethod in Classical and Computer Environments

To determine the list or (the table) of approximate values of the Euler's sequence terms, the students use the software Maple suggested by the official program suggested by the institution. This software makes it possible to perform the calculations of the numerical values of the terms of the sequence, to construct one or more approximate curves (Euler's curves), and to superimpose them in the same coordinate system. It also makes it possible to determine the exact solution in the case where the DE in question is accessible with an algebraic technique, and by superimposing its curve «exact» with approximate curves obtained with different values of the step h, the difference between the different curves and the "exact" one can be used to see the effect of the chosen step (or the number of subdivisions of the study interval). As shown in the following figure.

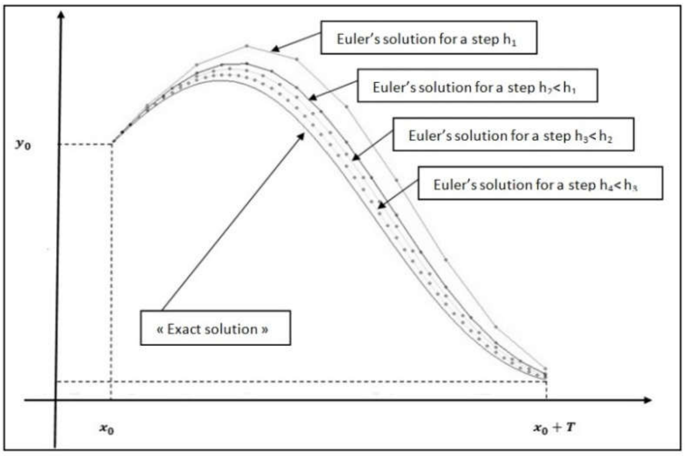


Figure 2. Euler curves obtained via Maple with different values of the step h

METHODOLOGY

Adaptation of the model of the dual didactic and computer transposition to the Euler method

Inspired by the double transposition model developed by Briant and Broner (2015), we identify three inseparable and complementary resolutions to solving a problem problem: that are inseparable and complementary: mathematic , algorithm and computing.

- The mathematical resolution that gives rise to a first algorithm : « mathematical algorithm ».
- The algorithmic resolution constitutes the first transposition: "to determine a computerized algorithm, written in pseudo-code".In some cases, the mathematical algorithm, usually used in the pencil-paper environment, requires knowledge of the mathematical object in question which is not generally implemented in any programming software.If the case arises, it is necessary to look for others that take into account the elementary actions that can be carried out by the machine or the integrated programming software.

- The computer resolution refers to the operation that results in writing the program with a computer language appropriate to the software in play, following the first transposition.

We summarize these types of resolutions related to the Euler's meth od using the double transposition process in the following table:

Mathematical resolution → Algorithmic resolution → Computer resolution

first-order DE with initial condition whose solution exists under certain conditions	Euler's sequence relating to the DE	Euler's algorithm relating to the ED	Program for calculating the terms of the Euler's sequence
Approximate with the Euler method the solution of the DE: $\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$ over the interval $[x_0 , x_0 + T]$	- Choose N the number of discretization steps of the interval $[x_0 , x_0 + T]$ - Define the discretization step $h = \frac{T}{N}$ - Define Euler sequences $\begin{cases} x_0 = x_0 \\ x_{k+1} = x_k + h \end{cases}$ And $\begin{cases} y_0 = y(x_0) \\ x_{k+1} = y_k + h.f(x_k, y_k) \end{cases}$	- Initialization of data T, N and h -Initialization of the initial conditions $x = x_0 , y = y_0$ -While $x \leq T$: do a-calculate $k = f(x,y)$ b-calculate $y = y + k * hx$ $x :== x + h$ c- save data	$T := \cdots ; N := \cdots ;$ $h := \frac{T}{N}$ $x := x_0 ; y := y_0$ For i from 0 to $N - 1$ do; $k := f(x,y)$ $y := y + k * hx;$ $x := x + h$; od; Print (x,y)

1[st] transposition → 2[nd] transposition

Table 2. The double transposition in the numerical resolution of an EDL by the Euler method

The first transposition « algorithmic » is done at different levels, at the level of language: " it is a question of moving from a mathematical language, that is to say the language usually used by mathematical writings" (Modeste 2012, p. 62) to a pseudo-code language similar to the programming language, free of its problems of syntax, at *the level of techniques* which differ from one algorithm to another, so the underlying technology-theories are then modified. The second transposition: Computer transposition is done at two levels, that of the language where it is a question of passing from the pseudo-code language of the algorithm to a computer language, that is to say a programming language, which requires reformulation to give an equivalent that is comprehensible by the machine, according to its internal structure and in its language and at *the level of the variables*, in this case, the mathematical variables used in the algorithms will give way to the computer variables in the computer program. These variables are part of the technology of computer praxeologies. The existence of algorithmic transposition is an important didactic aspect for understanding phenomena related to mathematical learning in general. This concept is intimately linked, asalready mentioned, to the mathematical organizations implemented at the level of learning practices. We want to collect information on how students manage both environments (pencil-paper and Maple) to accomplish the proposed tasks. How do they organize mathematical, algorithmic and computer work in the computer environment? What are the difficulties inherent in computer transposition?

Background to the experiment and data collection

Our experiment took place in the last quarter of the 2018-2019 academic year with first-year university students, eclectic engineering section. We analyze the written and digitized productions of these students pairs in response to a mathematical problem (Situation1 (in appendix) proposed by their workshop teacher as a control test. This problem is organized in two parts: a « theoretical p art » which consists in first solving the proposed DE in the pencil - paper environment, the objective being to evaluate the students' achievements in relation to the resolution of a first-order. And a « practical part » the purpose of which is to analyze the instrumented practices of the students through the stepsimplemented to solve this DE in the Maple environment. We emphasize the fact that most students at this time of the academic year have become familiar with this software by manipulating predefined functionalities and applying programs concerning notions of functions, derived, primitive, integral, suites...In addition, the test is conducted in a computer room equipped with computers. The students of the group are divided into 4 pairs which are allowed unrestricted access to the manipulation of this artifact. Students' pairs are asked to present their productions at the end of the session (1:30 a.m. duration), composed of written reports and digital files cont aining work done in the computer environment « Maple » and recorde d on machines.We analyze these productions by comparing the writte n traces with the digitized ones, targeting the three phases of resoluti ons implemented to solve the problem posed. For each stage of the tr ansposition, it is a question of identifying the techniques mobilized by the students and to identify the errors committed or the schemes developed. The digital approach via the Euler's method will require the coordination of graphic and algebraic and possibly analytical schemes related to the Maple environment. We show how these schemes are activated by students during this process. These schemes may occur in the representation register (discursive, graphic, algebraic, and oranalytical) and have different functions (Trouche (2005)) an heuristic function (control, organization of action), an pragmatic function (action, transformation) and an epistemic function (information gathering, understanding).

Analysis methodology

A priori analysis of the situation is conducted with reference to theoretical tools by considering two local mathematical organizations (LMOs) in the paper/pencil (p/c) environment relating to: (AR) algebraic resolution and (GR) graphical representation of the curve of the solution. We hypothesize that in the Maple environment the instrumentation process partially modifies these two LMOs,the techni ques and implementation procedures do not work in the same way as in the pencil -paperenvironment. We identify the possible strategies attached to instrumentation processes that could allow the modeling of the problem in Maple. We put forward two local mathematical organizations (LMOs), IGR: solution graphical representation and GRE: graphical representation of the solution by the Euler's method. Four criteria for GRE are identified:

C1: The program is correctly applied, the curve is plotted and t he development of the calculations is relevant.

C2: The minimum or approximate graphical representation an d the relevant algebraic calculations.

C3: The minimum graphical representation and irrelevant alge braic calculations.

C4: The graphical representation without the development of t he associated calculation.

It should be noted that the degree of elaboration of the graphical repr esentation by the Euler's method is the result of a correctly applied calculation program or algrithmic resolution. The interpretation of the curve, the numerical resolution of the differential equation and the approach of certain numerical values refer to the challenge that the graph represents in solving the problem.If the students make a relatively vagueor erroneous curve, it can only be an intuitive entry into the problem, then this curve is useless in setting up the numerical resolution of the differential equation or in interpreting the algebraic re solution. When the curve is constructed correctly, it refers to a correct application of the resolution program by the Euler method and a mastery of Maple's commands.The interpretation of the graph will b e an indication of the interaction between the graphical representation and the algebraic or numerical resolution of the DE involved.

MAIN RESULTS RELATING STUDENTS PRACTICES

The analysis of student practices allowed us to identify important elements for our problem.

In the Paper-Pencil Environment

The implementation of algebraic techniques seems to be automated without taking into account the information contained in the statement (nature of the DE, initial condition), which engages students in meaningless approaches. For example, the search for a particular solution by the constant variation method when the differential equation in play is homogeneous. The scheme developed by most students is that « any DE of the first order possesses a particular solution wich can be obtained by the method of variation of the constant ». a scheme that seems prag matic. These extracts from students productions illustrate our point.

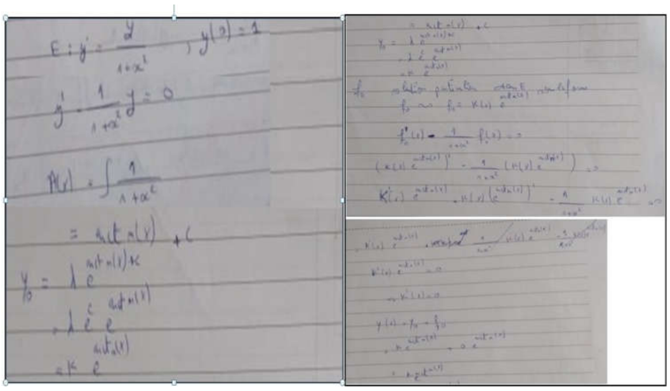


Fig 3 Extract from the production of pair 2

The work done in the graphic register is very poor and is not equipped with explicit technological elements around the function study which would allow a reasoned graphic representation of the solution function obtained. Although this first part is purely mathematical, the students use a computer resolution of the DE to benefit from the look of the solution that appears on the screen and reproduce it on their copies. The graphic scheme developed seems rather heuristic in nature. The computer transposition of the problem here does not allow to base a reflection on the technology underlying the graphic work. The computer resolution is initiated during the transposition process without being requested, as can be seen in this extract.

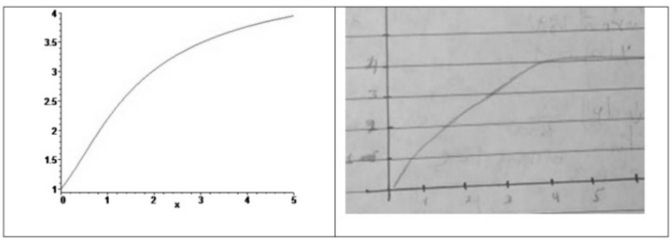


Fig 4 Extract from the production of pair 3

In addition, the traces found in the copies around the application of Euler's method via the algorithm institutionalized by the teacher reveal an insufficient mastery of the concepts involved in this technique, which explains the confusions committed by the students during the process of algorithmic transposition between mathematical variables and computer variables. These variables are conveyed by erroneously written ostensives that reflect the difficulties of conceptua lizing the non-ostensives to which they refer.The following extract illustrates some of these confusions.

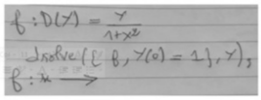
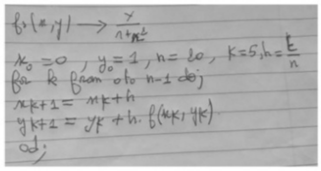
Confusion in the use of the ostensive "f", which refers to the solution of the DE and to the object DE as it, therefore between equation and function	Confusion between "f" the DE solution suggested by the statement and the two-variable function required by the application of the Euler method
	

Fig 5 Extract of production of pair 2

Moreover, there is no written record of an attempt to check the results found, which confirms the difficulties of the students in identifying cert ain mathematical objects in the same register and in perceiving the different semiotic representations according to the registers called by the resolution.As for the interpretation of the Euler curve as an approximation of the exact solutionin terms of step size and estimation errors, there is no trace in the students' productions. It seems that the mobilization of this algorithm is done in an automated manner without controlling the results by a return to the situation.

In the computer environment "Maple"

The analysis of the work accomplished by the students from the recordings made on the machine (digitized files), with regard to the written traces in the reports of the practical work carried out (copies) makes it possible to relate the procedures implemented in the passage from one resolution mode to another. Several observations are put forward. Students are having trouble producing a computer-resolution correctly The reproduction of the algorithm relating to the Euler method, which is supposed to be mobilized by the most of students at this time of the study (evaluation), is not self-evident and the results obtained in the pencil-paper environment reinforce this observation. Indeed, many confusions between ostensives reappear in the writing of the algorithm on the copies which refer to difficulties of conceptualizing algebraic expressions in general , the number of steps and amplitude called by the Euler method and, difficulties of epistemic nature related to the discrete and the continuous, as illustrated by the following extract.

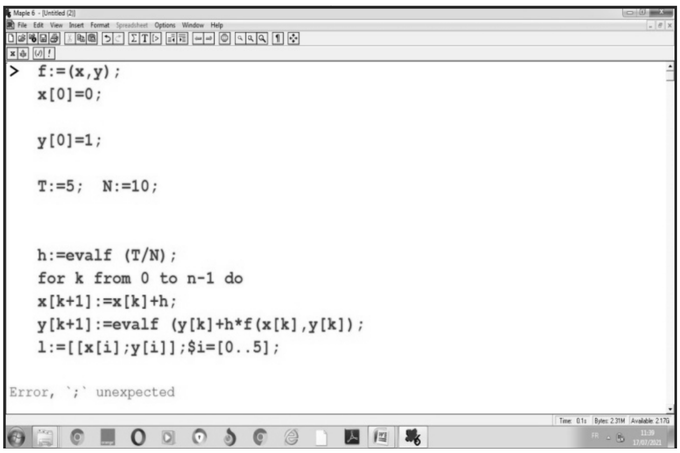


Fig 6. Extract from the production of pair 4

Students ignore feedback from Maple even when they receive an error message and continue to complete the requested task. When entering the list of Euler points, there are instructions that are independent, which seem to refer to insufficient mastery of the Euler method on the one hand and confusion between the variables: For the action which consists in declaring the list of Euler's points, we find the instruction ">l: = [[x [i]; y [i]]; $ i = [0..5] " ; instead of (> l: = [[x[i]; y[i]]$i = 0..N];) where N denotes the chosen number of steps. In the writing of pair 2 "$ i [0..5]; we perceive a confusion between the counter of iterations "i" ranging from 0 to 50 discrete and the real variable x belonging to the continuous interval [0.5]. We could hypothesize that the problem of the continuous and the discrete in the naturalized digital framework in the paper and pencil environment reappears in the writing of the algorithm and at the level of the computer transposition of Euler's method.

- The four pairs observed have difficulties in generalizing and interpreting the graphic plot, which suggests a dependence on the machine at the expense of a thoughtful int eraction between the two environments paper-pencil and computer. Difficulties also arise in interpreting the task and the dialectics between the numerical and graphic registers. Moreover, the set of frames called by the task also suggests difficulties in understanding the approximate calculation of the value of the solution function at a point, (being the main objective of Euler's method). It seems that the transition from the numerical (discrete) framework to the algebraic (continuous) framework is problematic and this is explicitly apparent in the computing environment when switching from algorithmic to computer transposition.

CONCLUSIONS AND DISCUSSIONS

The analyzes carried out within the framework of this research made it possible to put forward schemes developed by the students of an epistemic nature who revise an insufficient mastery of the numerical technique of resolution of the LDE and of the mathematical concepts involved in its implementation. The student practices seem, among other things, to respond to impoverished institutional practices in terms of graphic work and general modeling. The algebrai approach is present in teaching, but it is not operationalized for a study of the graphic or qualitative resolution of a differential equation. Most of the pairs observed did not manage to resolve graphically and then numerically the situation proposed for evaluation. Euler's method seems to be understood more as a resolution algorithm using a meaningless computer language for students to numerically solve a DE on the basis of precise mathematical procedures . This study through the analysis undertaken in the two environments, made it possible, to highlight conceptual difficulties for students that the paper-pencil environment alone would not have made it possible to detect via the written traces. For example, confusions between ostensives according to whether they refer to mathematical variables or to computer variable.Computer resolution also seems to allow better handling of graphic work from work equipped with instrumented techniques. This allows, on the one hand, to relieve the student of a task which may come to block the mathematical resolution of the problem but does not allow him to access an instrumental technique allowing a long-term instrumental genesis of the mathematical concept in play or of the recommended technique.This study examines the use of digital resources by actors in the education system, particularly teachers. The analysis of student practices suggests that the construction of algorithms comes at the end of mathematical concept learning processes, showing that implicitly the aim was mainly oriented towards a computer resolution through automated instrumented actions to the detriment of a consolidation of mathematical knowledge. Student practices also suggest that the teacher's didactic project during the workshop sessions is resolutely algorithmic, but the proposed tasks are based on emblematic mathematic knowledges requiring requiring passages in different conceptual frameworks in order to be able to handle algorithmic concepts to . This study also raises the difficulty for teachers of teaching knowledge under the guise of two reference disciplines (mathematics and computer science) and its effects on students and the often unreasonable use of digital resources made available to them within these workshops and as part of practical work in mathematics.

REFERENCES

ARTIGUE M.(1998). Un regard didactique sur l'utilisation d'outils de calcul formel dans l'enseignement des mathématiques. Repères IREM, 19 ;p. 77–108.

BALACHEFF N. (1994). Didactique et intelligence artificielle. Recherches en didactique des mathématiques 14(1),p. 9-42.

BEN NEJMA S. JABRANE, A (2021) . A Didactical Analysis of Teaching practices around Differential Equations in Tunisian School Context. (SJEDU). Science Journal of Education. Vol 9, NO 5 pp. 157-169. ISSN 2329-0900. ISSN Online 2329-0897- Science PG.

BEN NEJMA, S; JABRANE, A (2021). Une analyse de la transposition didactique des équations différentielles en terminale scientifique : réformes curriculaires et défis institutionnels- IJARTech International Journal of Applied Research and Technology Vol (N°3), 2021. ISSN 1737-7463- (PP 5-23-)

BRINSI L. (2020). La résolution numérique des équations différentielles dans un environnement informatique en première année d'université : pratiques enseignantes et genèse instrumentale des étudiants.Mémoire de master de recherche en didactiques des mathématiques. ISEFC-Université virtuelle de Tunis.

BRIANT N. & BRONNER A. (2015). Étude d'une transposition didactique de l'algorithmique au lycée : une pensée algorithmique comme un versant de la pensée mathématique. In Theis L. (Ed.) Actes du colloque EMF2015 – GT3 : Pluralités culturelles et universalité des mathématiques : enjeux et perspectives pour leur enseignement et leur apprentissage ;p. 231-246.

CHEVALLARD Y. (1985). La transposition didactique : du savoir savant au savoir enseigné, La pensée sauvage, Grenoble.

CHEVALLARD Y. (1999). Analyse des pratiques enseignantes en théorie anthropologique du didactique. Revue de didactique des mathématiques, 19 (2) ;p. 221-266. La pensée Sauvage, Grenoble.

CHEVALLARD Y. (1999). Analyse des pratiques enseignantes en théorie anthropologique du didactique. Revue de didactique des mathématiques, 19 (2) ; p. 221-266. La pensée Sauvage, Grenoble.

DUVAL, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la pensée. Annales de didactique et de sciences cognitives 5 ; p. 37-65. IREM de Strasbourg.

DUVAL R.(1995). Sémiosis et pensée humaine : Registres sémiotiques et apprentissages intellectuels. Berne : Peter Lang.

DUVAL R. (1996a). Quel cognitif retenir en didactique des mathématiques? Recherches en didactique des mathématiques 16 (3) ;p. 349–382.

DUVAL R. (1996b). Les représentations graphiques: fonctionnement et conditions de leur apprentissage», Actes de la 46ème Rencontre internationale de la CIEAEM, tome 1 ;p. 3–15 : Université Paul Sabatier, Toulouse.

MODESTE S. (2012). Enseigner l'algorithme pour quoi ? Quelles nouvelles questions pour les mathématiques ? Quels apports pour l'apprentissage de la preuve ? (Thèse de doctorat).Université de Grenoble.

RABARDEL P .(1995). Les hommes et les technologies. cognitive des instruments contemporain. Paris : Armand Colin

TROUCHE L. (2005). Construction et conduite des instruments dans les apprentissages mathématiques : nécessité des orchestrations. Recherches en Didactique des Mathématiques, 25,p. 91-138.

APPENDIX

Situation 1 :

(E) : $y' = \frac{y}{1+x^2}$, $y(0) = 1$

Partie théorique

1- Résoudre l'équation différentielle (E). On notera f la solution de (E).

2- Représenter l'allure de la courbe de f dans un repère orthonormé (O, I, J)

Partie pratique

1-En utilisant Maple, déterminer l'expression de $f(x)$ et une valeur approchée de $f(2)$.

2-Déterminer une valeur approchée de $f(2)$ avec la méthode d'Euler.

3-Construire C_1 la courbe de la fonction f sur l'intervalle[0,5].

4-En utilisant la méthode d'Euler et en choisissant un pas égale h= 0.5 construire la courbe C_2de la solution approchée de (E).

5-Représenter les courbes $C_1 et\ C_2$dans le même repère. Interpréter le résultat obtenu.

6-Refaire les questions 3/,4/et5/ avec un choix du pas h=0.1. Conclure.

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