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Research Article

THE NEW INTEGRAL TRANSFORM "HUNAIBER TRANSFORM"

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ABSTRACT

In this paper, we introduced a new integral transform namely Hunaiber transform. Proved operational properties of Hunaiber transform and presented the Hunaiber transform for some functions. Hunaiber transform was applied to solve linear ordinary differential equations with constant coefficients.

Keywords: Hunaiber Transform, Integral Transform, Differential Equations.

INTRODUCTION

Hunaiber Transform is derived from the classical Fourier integral. Hunaiber Transform was introduced by Mona Hunaiber to facilitate the process of solving differential equations. There are so many integral transforms has been developed for removing different differential operators, see in [1], [3], [4], [5], [6], [7], [9]. Fourier, Laplace, Elzaki,Aboodh, Tarig, Sumudu, Mahgoub, Kamal and Sadiktransforms are the convenient mathematical tools for solving differential equations. Also, Hunaiber transform and some of its fundamental properties are used to solve differential equations.

Definition.
Let $\psi(\gamma)$ be piecewise continuous on the interval $0 \le \gamma \le \kappa$ for any $\kappa > 0$, and$|\psi(\gamma)| \le Te^{a\gamma}$ where $\gamma \ge M$, for any real constant a and somepositive constants T and M. The Hunaiber transform denoted by the operator $\mathrm{H}(.)$is defined by

$$\mathrm{H}[\psi(\gamma)] = \Psi(\mu^\alpha, \beta) = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \psi(\gamma) d\gamma , \qquad (1.1)$$

where μ is complex variable, $\alpha \ne 0$ and β are real numbers. The operator H is called the Hunaiber transform operator.

HUNAIBER TRANSFORM OF ELEMENTARY FUNCTIONS

(1). If the function $\psi(\gamma) = 1$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} d\gamma = \frac{\mu^\beta}{\mu^\alpha} \qquad (2.1)$$

(2). If the function $\psi(\gamma) = \gamma$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \gamma d\gamma = \frac{\mu^\beta}{\mu^{2\alpha}} \qquad (2.2)$$

(3). If the function $\psi(\gamma) = \gamma^2$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \gamma^2 d\gamma = \frac{2\mu^\beta}{\mu^{3\alpha}} \qquad (2.3)$$

(4). If the function $\psi(\gamma) = \gamma^n$, $n = 0, 1, 2, \dots$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \gamma^n d\gamma = \frac{n!\, \mu^\beta}{\mu^{(n\alpha+\alpha)}} \qquad (2.4)$$

(5). If the function $\psi(\gamma) = e^{a\gamma}$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} e^{a\gamma} d\gamma = \frac{\mu^\beta}{\mu^\alpha - a} \qquad (2.5)$$

(6). If the function $\psi(\gamma) = \sin a\gamma$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \sin a\gamma \, d\gamma = \frac{\mu^\beta}{2i} \int_0^\infty e^{-\gamma\mu^\alpha} \left(e^{ai\gamma} - e^{-ai\gamma}\right) d\gamma = \frac{a\mu^\beta}{\mu^{2\alpha} + a^2} \qquad (2.6)$$

(7). If the function $\psi(\gamma) = \cos a\gamma$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \cos a\gamma \, d\gamma = \frac{\mu^\beta}{2} \int_0^\infty e^{-\gamma\mu^\alpha} \left(e^{ai\gamma} + e^{-ai\gamma}\right) d\gamma = \frac{\mu^\beta \mu^\alpha}{\mu^{2\alpha} + a^2} \qquad (2.7)$$

(8). If the function $\psi(\gamma) = \sinh a\gamma$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \sinh a\gamma \, d\gamma = \frac{a\mu^\beta}{\mu^{2\alpha} - a^2} \qquad (2.8)$$

(9). If the function $\psi(\gamma) = \cosh a\gamma$, then

$$\mathrm{H}[\psi(\gamma)] = \mu^\beta \int_0^\infty e^{-\gamma\mu^\alpha} \cosh a\gamma \, d\gamma = \frac{\mu^\beta \mu^\alpha}{\mu^{2\alpha} - a^2} \qquad (2.9)$$

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Theorem

Let $\Psi(\mu^{\alpha},\beta)$ *be the Hunaiber transform of the function* $\psi(\gamma)$*, then*

(i) $H[\psi'(\gamma)] = \mu^{\alpha}\Psi(\mu^{\alpha},\beta) - \mu^{\beta}\psi(0)$

(ii) $H[\psi''(\gamma)] = \mu^{2\alpha}\Psi(\mu^{\alpha},\beta) - \mu^{\alpha}\mu^{\beta}\psi(0) - \mu^{\beta}\psi'(0)$

(iii) $H[\psi^{(n)}(\gamma)] = \mu^{n\alpha}\Psi(\mu^{\alpha},\beta) - \mu^{(n-1)\alpha}\mu^{\beta}\psi(0) - \mu^{(n-2)\alpha}\mu^{\beta}\psi'(0) - \cdots - \mu^{\beta}\psi^{(n-1)}(0)$

Proof:

(i) By the definition (1.1),we have

$$H[\psi'(\gamma)] = \mu^{\beta}\int_0^{\infty} e^{-\gamma\mu^{\alpha}}\,\psi'(\gamma)d\gamma$$

Integrating by parts,we obtain

$$H[\psi'(\gamma)] = \mu^{\alpha}\,\Psi(\mu^{\alpha},\beta) - \mu^{\beta}\psi(0)$$

(ii) By the definition (1.1), we have

$$H[\psi''(\gamma)] = \mu^{\beta}\int_0^{\infty} e^{-\gamma\mu^{\alpha}}\,\psi''(\gamma)d\gamma$$

Integrating by parts,we obtain

$$H[\psi''(\gamma)] = \mu^{2\alpha}\Psi(\mu^{\alpha},\beta) - \mu^{\alpha}\mu^{\beta}\psi(0) - \mu^{\beta}\psi'(0)$$

(iii) Can be proof by mathematical induction.

APPLICATION OF HUNAIBER TRANSFORM OF ORDINARY DIFFERENTIAL EQUATIONS

As stated in the introduction of this paper, the Hunaiber transform can be used as an operative tool. For analyzing the basic characteristics of a linear system ruled by the differential equation in response to initial data. The following examples illustrate the use of the Hunaiber transform in solving certain initial value problems described by ordinary differential equations.

Consider the first-order ordinary differential equation

$$\frac{dy}{d\gamma} + py = \psi(\gamma), \qquad \gamma > 0 \tag{3.1}$$

with the initial condition

$$y(0) = a\,, \tag{3.2}$$

where a and p are constants.

Applying the Hunaiber transform to both sides of Eq. (3.1), we have

$$H\left[\frac{dy}{d\gamma} + py\right] = H[\psi(\gamma)] \tag{3.3}$$

Using the differential property of Hunaiber transform, Eq.(3.3) can be written as

$$\mu^{\alpha}H[y] - \mu^{\beta}y(0) + pH[y] = \Psi(\mu^{\alpha},\beta) \tag{3.4}$$

Using the initial condition(3.2)gives

$$H[y] = \frac{\Psi(\mu^{\alpha},\beta) + a\mu^{\beta}}{\mu^{\alpha} + p} \tag{3.5}$$

The inverse Hunaiber transform leads to the solution.

Consider the second-order linear ordinary differential equation has the general form

$$\frac{d^2y}{d\gamma^2} + 2p\frac{dy}{d\gamma} + qy = \psi(\gamma), \qquad \gamma > 0 \tag{3.6}$$

with the initial conditions

$$y(0) = a \quad , \quad y'(0) = b\,, \tag{3.7}$$

where $a,\ b,\ p$ and q are constants.

Applying the Hunaiber transform to both sides of Eq.(3.6), we have

$$H\left[\frac{d^2y}{d\gamma^2} + 2p\frac{dy}{d\gamma} + qy\right] = H[\psi(\gamma)] \tag{3.8}$$

Using the differential property of Hunaiber transform, Eq. (3.8)and using the initial conditions, we obtain

$$\mu^{2\alpha}H[y] - \mu^{\alpha}\mu^{\beta}y(0) - \mu^{\beta}y'(0) + 2p\mu^{\alpha}H[y] - 2p\mu^{\beta}y(0) + qH[y] = \Psi(\mu^{\alpha},\beta) \tag{3.9}$$

Using the initial conditions (3.7), we get

$$H[y] = \frac{\Psi(\mu^{\alpha},\beta)}{\mu^{2\alpha} + 2p\mu^{\alpha} + q} + \frac{a\mu^{\beta}(\mu^{\alpha} + 2p)}{\mu^{2\alpha} + 2p\mu^{\alpha} + q} + \frac{b\mu^{\beta}}{\mu^{2\alpha} + 2p\mu^{\alpha} + q} \tag{3.10}$$

The inverse Hunaiber transform leads to the solution.

Example 1. *Consider the first order differential equation*

$$\frac{dy}{d\gamma} - 2y = 0\,, \qquad y(0) = 1 \tag{3.11}$$

Take Hunaiber transform to this equation gives

$$\mu^{\alpha}H[y] - \mu^{\beta}y(0) + 2H[y] = 0\,, \tag{3.12}$$

whereH is the Hunaiber transform of the function $y(\gamma)$.

Applying the initial condition, we get

$$(\mu^{\alpha} + 2)H[y] = \mu^{\beta} (3.13)$$

$$H[y] = \frac{\mu^{\beta}}{\mu^{\alpha} + 2} \tag{3.14}$$

Now, applying the inverse Hunaiber transform, we get

$$y(\gamma) = H^{-1}\left[\frac{\mu^{\beta}}{\mu^{\alpha} + 2}\right] = e^{-2\gamma} \tag{3.15}$$

Example2. *Consider the second-order differential equation*

$$\frac{d^2y}{d\gamma^2} - 3\frac{dy}{d\gamma} + 2\,y = 0\,, \qquad y(0) = 1\,,\ y'(0) = 4 \tag{3.16}$$

Applying the Hunaiber transform to both sides of the equation gives

$$H[y] = \frac{\mu^{\beta}(\mu^{\alpha} - 3)}{\mu^{2\alpha} - 3\,\mu^{\alpha} + 2} + \frac{4\,\mu^{\beta}}{\mu^{2\alpha} - 3\,\mu^{\alpha} + 2} \tag{3.17}$$

$$H[y] = \frac{3\mu^{\beta}(\mu^{\alpha} - 1)}{(\mu^{\alpha} - 1)(\mu^{\alpha} - 2)} + \frac{4\,\mu^{\beta} - 2\mu^{\beta}\mu^{\alpha}}{(\mu^{\alpha} - 1)(\mu^{\alpha} - 2)} \tag{3.18}$$

$$\mathrm{H}[y] = \frac{3\mu^{\beta}}{\mu^{\alpha}-2} - \frac{2\mu^{\beta}}{\mu^{\alpha}-1} \tag{3.19}$$

Inverting to find the solution in the form

$$y(\gamma) = 3\, e^{2\gamma} - 2\, e^{\gamma} \tag{3.20}$$

Example 3.*Consider the second-order differential equation*

$$\frac{d^2y}{d\gamma^2} + 4\, y = 12\, \gamma\,, \quad y(0) = 0, \quad y'(0) = 7 \tag{3.21}$$

Applying the Hunaiber transform to both sides of the equation gives

$$\mathrm{H}[y] = \frac{12\mu^{\beta}}{\mu^{2\alpha}(\mu^{2\alpha}+4)} + \frac{7\mu^{\beta}}{\mu^{2\alpha}+4} \tag{3.22}$$

$$\mathrm{H}[y] = \frac{12\mu^{\beta} + 3\mu^{\beta}\mu^{2\alpha}}{\mu^{2\alpha}(\mu^{2\alpha}+4)} + \frac{4\mu^{\beta}}{\mu^{2\alpha}+4} \tag{3.23}$$

$$\mathrm{H}[y] = \frac{3\mu^{\beta}}{\mu^{2\alpha}} + \frac{4\mu^{\beta}}{\mu^{2\alpha}+4} \tag{3.24}$$

Inverting to find the solution in the form

$$y(\gamma) = 3\gamma + 4\, sin\, 2\gamma \tag{3.25}$$

CONCLUSION

In conclusion, There are a lot of the integral transforms of exponential type kernels, the Hunaiber transform is new and very powerful among them. Definition, application of the new transform "Hunaiber transform" to the solution of ordinary differential equations has been demonstrated.

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