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Research Article

ON SD-CONNECTED SPACES

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ABSTRACT

In this article, we introduce a stronger form of connected space using the notion of somewhere dense sets, namely SD-connected space. Firstly, we define SD-separated sets in topological spaces,then we discuss their characterizationsand use their notions to study the behaviour of SD-connected space. Moreover, we discuss the concept of SD-connected sets and SD-connected subspaces, where we show that these concepts are different. Additionally, images of SD-connected space are established under some particular maps as SD-irresolute maps and SD-continuous maps. Finally, we prove that any somewhere dense set is dense inSD-connected space, also we illustrate the relationships between this class of spaces and strongly hyperconnected space, hyperconnected space and connected space, and we show that these spaces are strongly ordered as: strongly hyperconnected space, SD-connected space, hyperconnected space and then connected space.

Keywords:Topological space and generalizations,somewhere dense sets, subspaces, strongly hyperconnected space, connectedness.

AMS Subject Classification (2000): 54A05, 54B05, 54D05.

INTRODUCTION

Topologists generalized many concepts in general topology such as continuity, compactness, connectedness, etc. by using some different types of generalized open sets as; α-open set, pre-open set, semi-open set, b-open set, β-open set, somewhere dense set, etc.[1-7]. Separated and connectedness have been studied by many researchers based on different generalized open sets such as α-connected space, pre-connected space, semi-connected space, b-connected space and β-connected space[8-11].In 2017 [12], Al-shami introduced and studied the concepts of somewhere dense sets and ST_1-spaces, few years later, Al-shami and Noiri [13] have studied further properties of somewhere dense sets, and have defined some maps in terms of somewhere dense sets asSD-continuous and SD-irresolute maps, moreover, they defined the notion of SD-cover and use it to introduced compactness and Lindelöfness via s-dense sets in [14].Recently, Arwini and Mira [15] proved that the space is strongly hyper connected if and only if open sets and somewhere dense sets are coincide.

The main goal of this article, is to extend Al-shami's work and introduce a new type of connectedness, by using the concept of somewhere dense sets, we call it SD-connected space, then we discuss its properties and show that this space is stronger than connected space, but weaker than strongly hyper connected space, additionally, we investigate their characterization and study their images by some particular maps, finally, we prove that any somewhere dense set is dense in SD-connected space, and this fact implies that SD-connected space is stronger than hyperconnected space.

This article is divided to eight sections, as follows: after presenting the introduction, we review some preliminaries in section two, where we recall some definitions and results that related to somewhere dense sets, hyperconnected spaces, strongly hyperconnected spaces, SD-maps and some types of connected spaces. In section three, we define SD-separated sets in topological spaces using the notions of somewhere dense sets, and then we study their relation with separated sets. SD-connected spaces are introduced in section four, where we used the notion of SD-separated sets. Section five conclude the concepts of SD-connected sets, and SD-connected subspaces, where we show that these concepts are different. Images of SD-connected spaces have been studied by some maps as SD-irresolute maps and SD-continuous maps in section six. In section seven, we investigate the characterization of SD-connected spaces and establish their implication with strongly hyperconnected space, hyperconnected space and connected spaces. Finally, in section eight, we summarize our results in the conclusion.

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PRELIMINARIES

In this section, we recall some definitions and results that related to somewhere dense sets, hyperconnected spaces, strongly hyper connected spaces, SD-maps and some types of connected spaces, that we need in our study.Throughout thepresent article,(X, τ)or simply Xrepresents topological space which no separation axioms are assumed, unless otherwise are mentioned. The closure and the interior of a subset A of a space X are denoted by$\overline{A}$ and A^o; respectively, and the complement of the set Ain X, the difference of A and B, and the power set of X are denoted byA^c, $A/B$and $P(X)$; respectively. Moreover,the sets $\mathbb{R}$,$\mathbb{Q}$ and $\mathbb{K}$denote the real numbers, rational numbers and irrational numbers; respectively.

Definition 2.1.[12] A subset B of a topological space (X, τ) is called somewhere dense (briefly s-dense) if the interior of its closure is non-empty, i.e. $\overline{B}^o \neq \phi$. The complement of s-dense set is called closed somewhere dense (briefly cs-dense), and the collection of all s-dense sets in X is denoted by $S(\tau)$.

Corollary 2.2. [12] In a topological space (X, τ), we have:

- any open set is s-dense.

- any dense set is s-dense.
- any set in X that contains a s-dense set is s-dense.

Theorem 2.3. [12] Every subset of a space $(X, \tau)$is s-dense or cs-dense.

Definition 2.4. [14] A subset B of a space (X, τ) is called SD-clopen (SC-set) if$B$ is s-dense and cs-dense.

Definition 2.5.[12] Let A be a subset of a space (X, τ), then the S-closure of A (briefly Scl(A) or$\overline{A}^S$)is the intersection of all cs-dense sets containing$A$.

Corollary 2.6. [12]If M is a subset of a space(X, τ), then $M \subseteq \overline{M}^S \subseteq \overline{M}$.

Theorem 2.7.[15] Let (X, τ) be a topological space, Y be a subspace of$X$ and$A \subseteq Y$then:

1. If Y is closed and A is s-dense in X, then A is s-dense in Y.
2. If Y is open and A is s-dense in Y, then A is s-dense in X.

Definition 2.8.[12] A topological space (X, τ) is calledST_1-space if for any pair of distinct points $a, b \in X$ there exist two s-dense sets one containing a but not b and theother containing b but not a.

Corollary 2.9.[12] A space$X$ isST_1-spaceif each pair of distinct points $a, b \in X$ can be separated by two disjoint s-dense sets.

Definition 2.10.[16] A subset B of a space (X, τ) is called regular open (briefly r-open) if $B = \overline{B}^o$, while the complement of regular open set is called regular closed (briefly r-closed). The family of all r-open sets in X is denoted by $RO(X, \tau)$.

Corollary 2.11. [16] Every r-open set is open.

Definition 2.12.[5] A topological space X is called partition space if every open subset of X is closed.

Definition 2.13.[17,18] A topological space$X$is called:

- Submaximal if any dense set in X is open.
- hyperconnected if any non-empty open set in X is dense.

Definition 2.14.[12] A subset A of a topological space (X, τ) is called:

- β-open set if $B \subseteq \overline{\overline{B}^o}$.
- b-open set if $B \subseteq \overline{B}^o \cup \overline{B^o}$.
- pre-open set if $B \subseteq \overline{B}^o$.
- α-open set if $B \subseteq \overline{B^o}^o$.

Corollary 2.15.[12]In a space X,α-open sets, pre-open sets, b-open sets, β-open and s-dense sets are all weaker than open sets, and they are strongly ordered as: open sets, α-open sets, pre-open sets, b-open sets, β-open sets and s-dense sets.

Theorem 2.16.[13] In hyperconnected space X, s-dense sets and β-open sets are coincide.

Definition 2.17.[15] A topological space $(X, \tau)$is called strongly hyperconnected if$X$ is submaximal and hyperconnected space, equivalently; non-empty open sets are coincide with dense sets.

Theorem 2.18.[15]A space $(X, \tau)$is strongly hyperconnectedspace iffany s-dense set in X is open.

Definition 2.19.[13]A map $F: (X, \tau) \to (Y, \theta)$ is called:

- SD-continuous map if the inverse image of each open subset of (Y, θ) is empty or s-dense subset of X.
- SD-irresolute map if the inverse image of each somewhere dense subset of Y is empty or a somewhere dense subset of X.

Theorem 2.20.[13] If $F: (X, \tau) \to (Y, \theta)$ is a SD-continuous map then the inverse image of each closed subset of (Y, θ) is X or cs-dense.

Definition 2.21. [19]Subsets$A$ and B of a space$(X, \tau)$are called separated if $B \cap \overline{A} = A \cap \overline{B} = \emptyset$.

Definition 2.22. [8] A topological space (X, τ) is connected (α-connected, pre-connected, b-connected, β-connected)if X can not be written in the form $X = A \cup B$; where$A$ and B are disjoint non-empty open (α-open, pre-open, b-open, β-open)sets in$X$.

Theorem 2.23.[8]Connected, α-connected, pre-connected, b-connected and β-connected spaces are strongly ordered as: β-connected space, b-connected space, pre-connected space, α-connected space and connected space.

SD-SEPARATED SETS

In this section, we define the notion of SD-separated sets in topological spaces, and study their properties and show how they relate to separated sets.

Definition 3.1. Let(X, τ) be atopological space, and let A and Bbe subsets of$X$,then A and B are called SD-separated if it is disjoint from the S-closure of the other, that is: $B \cap \overline{A}^S = A \cap \overline{B}^S = \emptyset$.

Corollary 3.2. In a space X:

- If A and B are separated sets, then$A$and B are SD-separated.
- Any SD-separated sets are disjoint.
- Subsets of SD-separated sets are SD-separated, i.e. if A and B are SD-separated and$C \subseteq A$ and $D \subseteq B$, then C and D are SD-separated.

Proof.

- Direct since $\overline{A}^S \subseteq \overline{A}$ and $\overline{B}^S \subseteq \overline{B}$.
- Direct since $A \subseteq \overline{A}^S$ and $B \subseteq \overline{B}^S$.
- Since $C \subseteq A$ and $\overline{D}^S \subseteq \overline{B}^S$,we have $C \cap \overline{D}^S \subseteq A \cap \overline{B}^S = \emptyset$.Similarly $D \cap \overline{C}^S = \emptyset$.

Examples 3.3.

1. In the trivial topologyon $\mathbb{R}$, we have $S(\tau) = P(\mathbb{R})/\{\emptyset\}$. So the sets $\mathbb{Q}$ and $\mathbb{K}$are SD-separated but not separated, since$\overline{\mathbb{Q}}^S = \mathbb{Q}$ and$\overline{\mathbb{K}}^S = \mathbb{K}$, but $\overline{\mathbb{Q}} = \overline{\mathbb{K}} = \mathbb{R}$.
2. In the space (X, τ), where $X = \mathbb{R}$ and $\tau = \{V \subseteq \mathbb{R}: 0 \notin V\} \cup \{\mathbb{R}\}$, we have $S(\tau) = P(\mathbb{R})/\{\{0\}, \emptyset\}$.Then the sets $\{0\}\ and\ \{0\}^C$ are disjoint but not SD-separated, since $\overline{\{0\}^C}^S = \mathbb{R}$.
3. In any partition space (X, τ), any disjoint sets are SD-separated, since any non empty subset of X is s-dense, i.e. $S(\tau) = P(X)/\{\emptyset\}$.

Theorem3.4. If(X, τ)be a topological space, then the following statement are equivalent:

1. $X = A \cup B$, where A and B are disjoint s-dense sets.

2. $X = A \cup B$, where A and B are disjoint cs-dense sets.
3. $X = A \cup B$, where A and B are non-empty SD-separated sets.

Proof.

$(1) \Longrightarrow (2)$ Since $X = A \cup B$ and $A \cap B = \emptyset$, then $A = B^C$ and $B = A^C$, so A and B are cs-dense sets, and both A and B are proper non-empty sets of X, then A^C and B^C are non-emptycs-dense sets.

$(2) \Longrightarrow (3)$SinceX $= A \cup B$, where A and B are disjoint cs-dense sets, we have $A \cap \overline{B}^S = A \cap B = \emptyset$, and $B \cap \overline{A}^S = B \cap A = \emptyset$, henceA and B are non-empty SD-separated.

$(3) \Longrightarrow (1)$ If $X = A \cup B$, where A and B are non-empty SD separated sets, then $A \cap \overline{B}^S = B \cap \overline{A}^S = \emptyset$,since $A \cap B = \emptyset$, $B \subseteq \overline{B}^S$and $A \subseteq \overline{A}^S$, then $B = \overline{B}^S$ and $A = \overline{A}^S$, so both A and B are non-emptycs-dense sets, but $A = B^C$and $B = A^C$, henceA and B are disjoint s-dense sets.

SD-CONNECTED SPACES

SD-connected space have been introduced in this section, by using the notion of somewhere dense sets, then we prove that this space is stronger than connected space.

Definition 4.1.A spaceX is called SD-connected space, if X cannotbe written in the formX $= A \cup B$; where A and B are disjoint s-dense sets in X.

Theorem 4.2. Any SD-connected space is connected.

Proof. Direct since any open set is s-dense.

SD-Connected Spaces⟹ Connected Spaces

Examples 4.3.

1. The trivial space (X,τ), where X has more than one element is connected but not SD-connected, since any non-empty subset of $X$is s-dense, therefore$X = A \cup A^C$; for any proper non-empty subset A of X.
2. In example (3.3.(2)), the space $(\mathbb{R},\tau)$ is connected but not SD-connected, since$\mathbb{R} = \{1\} \cup \{1\}^C$and $\{1\},\{1\}^C$ are disjoint s-dense sets.
3. Any partition space X with more than one element is neither SD-disconnected nor disconnected.

Theorem 4.4. In any space (X,τ), the following statements are equivalent:

1. X is SD-connected space.
2. X is not the union of two disjoint s-dense sets.
3. X is not the union of two disjoint cs-dense sets.
4. X is not the union of two non-empty SD-separated sets.

Proof. Direct from theorem(3.4).

Theorem 4.5.A topological space$(X,\tau)$is SD-connected iff there is no SD-clopen set inX.

Proof. Suppose A is a SD-cl open set in X, then $X = A \cup A^C$, where A andA^C are both s-dense, so X is SD-disconnected.Conversely, suppose X is SD-disconnected, then $X = A \cup B$, where A and B are disjoint s-dense in X so $B = A^C$, thereforeA is a SD-clopen set in X.

Lemma 4.6. [14]Any proper non-emptyr-open set in X is SD-clopen.

Proof. Suppose A is a non-empty proper r-open set, then$\overline{A}^o = A$, so $\overline{A} \neq X$, hence $\overline{A}^C \neq \emptyset$ and we have$A \subseteq \overline{A}$ so $\overline{A}^C \subseteq A^C$and since $\overline{A}$is closed, we obtain$\overline{A}^C$ is open so it is s-dense and since$\overline{A}^C \subseteq A^C$, therefore$A^C$ is s-dense, we get A is SD-clopen.

Remark 4.7. Any proper non-empty r-closed set is SD-clopen.

Proof. If B isproper non-empty r-closed in a space X, then B^C is r-open, so from the previous lemmaB^C is SD-clopen, hence B is SD-clopen.

Theorem 4.8. If $(X,\tau)$is SD-connected space, then $RO(X,\tau) = \{X,\emptyset\}$.

Proof. Suppose A is a non-empty proper r-open set in X, then A is SD-clopen, hence X is SD-disconnected, which contradict the assumption.

Corollary 4.9. Any topological space that has a non-empty proper r-open set (i.e. $RO(X,\tau) \neq \{X,\emptyset\}$) is SD-disconnected.

Example 4.10. The inverse of the pervious corollary is not true in general, for example: If $X = \mathbb{R}$ and τ is the trivial space on $\mathbb{R}$, then $RO(X,\tau) = \{X,\emptyset\}$, but X is SD-disconnected.

Theorem 4.11. IfX isST_1 space where X has more than one element; then X is SD-disconnected.

Proof. Suppose x and yare two distinctpoints in X, then there are disjoint s-dense sets A and B such that $x \in A$ and$y \in B$,then $B \subseteq A^C$, so A^C is also s-dense, hence A is SD-clopen,i.e.X is SD-disconnected.

Theorem 4.12. Any SD-connected space is β-connected (b-connected, pre-connected, α-connected) space.

Proof. Direct from theorem (2.23) and corollary (2.15), and these spaces are strongly ordered as:

SD-connected space⟹β-connected space⟹ b-connected space⟹pre-connected space⟹conneced space

SUBSPACES OF SD-CONNECTED SPACES

In this section, we give a definition of SD-connected sets, and show that the concepts of SD-connected sets and SD-connected subspace are different in general.

Definition 5.1.A subset C of a topological space (X,τ) is called SD-connectedset if we cannot write C in the form $C = A \cup B$; where A and B are disjoint s-dense sets in X.

Theorem 5.2. Any subset of SD-connected space is SD-connected set.

Proof. Suppose C is SD-disconnected subset of a space X, then $C = A \cup B$; where A and B are disjoint s-dense sets in X. Since$B \subseteq A^C$, we haveA^C is s-dense in XandX $= A \cup A^C$, hence X is SD-disconnected space.

Examples 5.3.

1. If X is aspace, then any singleton is SD-connected set in X.
2. If $X = \mathbb{R}$, $\tau = \{V \subseteq \mathbb{R}: 0 \notin V\} \cup \{\mathbb{R}\}$, then$\mathbb{R}$ is SD-disconnected space but the sets$\{0\}$ and $\{0,1\}$ are SD-connected sets, while $\{0,1,2\}$ is SD-disconnected set.

Remark 5.4. If A and B are SD-connected sets and $A \cap B \neq \emptyset$, then $A \cup B$ need not be SD-connected in general, for example: in example (5.3.(2)) the sets $\{0,1\}$ and $\{0,2\}$ are SD-connected sets, but

$\{0,1,2\}$ is SD-disconnected set, since $\{0,1\}$ and $\{2\}$ are both s-dense in $\mathbb{R}$.

In next results, we point out when the relative topology is SD-connected space:

Examples 5.5.

1. If $X = \mathbb{R}$, $\tau = \{V \subseteq \mathbb{R}: 0 \notin V\} \cup \{\mathbb{R}\}$, then $\{0,1\}$ is SD-connected set and the relative topology on $\{0,1\}$ is also SD-connected space.
2. If Y is SD-connected set $\nRightarrow (Y, \tau_Y)$ is SD-connected space; for example: If $X = \mathbb{R}$, $\tau = \{V \subseteq \mathbb{R}, 0 \in V\} \cup \{\emptyset\}$, then X is SD-connected space, but the relative topology on $Y = \{0\}^C$ is the discrete topology on Y, so it is SD-disconnected space, while the subset Y is SD-connected set.

Theorem 5.6. If (X, τ) is SD-connected space, and Y is an open subset of X, then the relative topology (Y, τ_Y) is SD-connected space.

Proof. Let (X, τ) be SD-connected space and Y be an open subspace of X. Suppose that Y is SD-disconnected space, then Y can be written as $Y = A \cup (Y/A)$, where A and (Y/A) are s-dense sets in Y. Since A and (Y/A) are s-dense sets in open subspace, then from theorem (2.7.(2)) A and (Y/A) are s-dense in X. So $X = A \cup (X/A)$, since (Y/A) is s-dense in X and $(Y/A) \subseteq (X/A)$, then by corollary (2.2.(iii)) we obtain (X/A) is also s-dense in X, so X is SD-disconnected, that contradict the assumption. Hence Y is SD-connected space.

Example 5.7. In the space $(\mathbb{R}, \tau)$ where $\tau = \{A \subseteq \mathbb{R}: 0 \in A\} \cup \{\emptyset\}$, we have $S(\tau) = \tau$. Then the space $\mathbb{R}$ is SD-connected but the subspace $\{0\}^C$ is SD-disconnected space.

Theorem 5.8. Let (X, τ) be a topological space and Y be open SD-connected set in X, then the relative topology (Y, τ_Y) is SD-connected space.

Proof. Suppose Y is SD-disconnected space, then $Y = A \cup B$; where A and B are disjoint s-dense in Y. Since Y is open, then from theorem (2.7.(2)) A and B are disjoint s-dense in X, so Y is SD-disconnected set.

Theorem 5.9. Let (X, τ) be a topological space and Y be a closed subset of X, where (Y, τ_Y) is SD-connected space, then Y is SD-connected set in X.

Proof. Suppose Y is SD-disconnected set, then $Y = A \cup B$; where A and B are disjoint s-dense in X. Since $A, B \subseteq Y \subseteq X$ and Y is closed, then from theorem (2.7.(3)) A and B are s-dense in Y; hence (Y, τ_Y) is SD-disconnected space.

IMAGES OF SD-CONNECTED SPACES

Here we study the images of SD-connected spaces by some particular maps defined in termes of somewhere dense sets, as SD-irresolute map and SD-continuous maps.

Theorem 6.1. The image of SD-connected space by SD-irresolute map is SD-connected set.

Proof. Let $F: X \to Y$ be a SD-irresolute map from SD-connected space X into a space Y. Suppose $F(X)$ is SD-disconnected set in Y, then $F(X) = A \cup B$; where A and B are disjoint s-dense sets in Y. Since F is SD-irresolute, we have $X = F^{-1}(A) \cup F^{-1}(B)$, where $F^{-1}(A)$ and $F^{-1}(B)$ are disjoint s-dense sets in X, hence X is SD-disconnected, which contradict the assumption.

Corollary 6.2. If $F: X \to Y$ is a SD-irresolute map from SD-connected space X onto a space Y, then Y is SD-connected space.

Proof. Similar proof as in the previous theorem where $F(X) = Y$.

Theorem 6.3. Let $F: X \to Y$ be a SD-irresolute map from SD-connected space X into a space Y. If $F(X)$ is open subspace of Y, then the subspace $(F(X), \tau_{F(X)})$ is SD-connected space.

Proof. Suppose $(F(X), \tau_{F(X)})$ is SD-disconnected space, then $F(X) = A \cup B$; where A and B are disjoint s-dense sets in $F(X)$. Since $F(X)$ is open subspace in Y then from theorem (2.7.(2)) A and B are disjoint s-dense sets in Y, so $F(X)$ is SD-disconnected set in Y, which contradict theorem (6.1).

Theorem 6.4. The image of SD-connected space under SD-continuous map is connected space.

Proof. Let $F: X \to Y$ be an SD-continuous map from SD-connected space X into a space Y, and suppose $F(X)$ is disconnected space, then $F(X) = A \cup B$; where A and B are non-empty disjoint open sets in Y, since $F(X)$ is SD-continuous, we obtain $X = F^{-1}(A) \cup F^{-1}(B)$; where $F^{-1}(A)$ and $F^{-1}(B)$ are disjoint s-dense sets in X, hence X is SD-disconnected which is a contradiction.

Example 6.5. The image of SD-connected space need not be SD-connected space under SD-continuous map, for example: Let $X = Y = \mathbb{R}$ and $\tau = \{V \subseteq \mathbb{R}, 0 \in V\} \cup \{\mathbb{R}\}$ while σ is the trivial topology in $\mathbb{R}$, hence the identity map $I: (\mathbb{R}, \tau) \to (\mathbb{R}, \sigma)$ is SD-continuous from SD-connected $(\mathbb{R}, \tau)$; while $(\mathbb{R}, \sigma)$ is SD-disconnected space.

Theorem 6.6. A topological space X is SD-connected iff every SD-continuous map $F: X \to \{0,1\}$ is constant.

Proof. From the previous theorem, we have $F(X)$ is connected subspace in $\{0,1\}$ so $F(X) = \{1\}$ or $F(X) = \{0\}$, i.e. F is constant. Conversely, if X is SD-disconnected and B is SD-clopen set in X, then the characteristic function $\chi_B: X \longrightarrow \{0,1\}$ is SD-continuous but not constant.

PROPERTIES OF SD-CONNECTED SPACES

This section provides some properties of SD-connected spaces, and the implication between this space with some known spaces as; strongly hyperconnected space, hyperconnected space and connected space.

Lemma 7.1. If (X, τ) is a topological space and $B \subseteq X$ such that $B^o = \emptyset$, then B^C is dense.

Proof. Let V be an open set such that $V \cap B^C = \emptyset$, then $V \subseteq B$, and since $B^o = \emptyset$, we have $V = \emptyset$. Hence B^C is dense in X.

Theorem 7.2. In SD-connected space X any s-dense set is dense.

Proof. Suppose A is a proper s-dense subset of SD-connected space X. Then $\overline{A}^0 \neq \emptyset$ and $A \neq X$, therefore $A^C \neq \emptyset$, and since X is SD-connected space, A^C cannot be s-dense, so $\overline{A^C}^0 = \emptyset$, then $\overline{A^C}^C$ is dense and $\overline{A^C}^C \subseteq A$, hence A is dense in X.

Example 7.3. The inverse of the previous theorem is not true in general, for example: If $X = \mathbb{R}$ and $\tau = \{\mathbb{R}\} \cup P(\mathbb{Q})$, then $S(\tau) = D(\tau) = \{A \subseteq \mathbb{R}: A \cap \mathbb{Q} \neq \emptyset\}$; where $D(\tau)$ is the collection of all dense sets in X. Therefore s-dense and dense sets are coincide but the space $\mathbb{R}$ is SD-disconnected, since $\mathbb{R} = \mathbb{Q}^+ \cup (\mathbb{Q}^- \cup \mathbb{K})$ where $\mathbb{Q}^+$ and $\mathbb{Q}^- \cup \mathbb{K}$ are both s-dense sets in $\mathbb{R}$. (The sets $\mathbb{Q}^+$, $\mathbb{Q}^-$ denote the positive and negative rational numbers; respectively).

Theorem 7.4. Any strongly hyperconnected space is SD-connected.

Proof. SupposeXstrongly hyperconneced space but SD-disconnected space, then there exists a SD-clopen set Aof X,where$A \neq \emptyset$and$A \neq X$,sinceXis strongly hyperconnected space and from theorem (2.18), then A and A^Carenon-empty open sets, so A is not dense (since$A \cap A^C = \emptyset$), hencethe setA is a non-empty open set but not dense, and thatcontradict definition (2.17).

Strongly hyperconnected space ⟹ SD-connected space

Example 7.5.SD-connected space need not bestrongly hyperconnected, for example: Let $X = \mathbb{R}$ and $\tau = \{\mathbb{R}, \emptyset, \{0\}\}$, then $S(\tau) = D(\tau) = \{A \subseteq \mathbb{R}: 0 \in A\} \cup \{\emptyset\}$. Note that $(\mathbb{R}, \tau)$ is SD-connected space, but not strongly hyperconnected, since $\{0,1\}$ is s-dense in X but not open.

Theorem 7.6. Any SD-connectedspace ishyperconnected.

Proof. LetXbe an SD-connectedspace, and let Vbe a non-empty open set in X, then V is s-dense and fromtheorem (7.2) we obtainV is densein X,henceX is hyperconnected space.

SD-connected space ⟹Hyperconnected space

Example 7.7. Hyperconnected spaceneed not beSD-connected in general, for example:If$X = \{1,2,3\}$ and $\tau = \{X, \emptyset, \{1,2\}\}$, then $S(\tau) = \{X, \emptyset, \{1,2\}, \{1\}, \{2\}, \{1,3\}, \{2,3\}\}$, so X is hyperconnected space, but not SD-connected, since $X = \{1\} \cup \{2,3\}$; where $\{1\}$ and $\{2,3\}$ are both s-dense sets in X.

Theorem 2.8. In hyperconnected space X, these condition are equivalently:

- $X$is SD-connected space.
- $X$is β-connected space.

Proof. Direct from theorem (2.16)since in hyper connected space s-dense sets and β-sets are coincide.

Theorem 2.9. In strongly hyperconnected space X, these condition are equivalently:

- $X$is SD-connected space.
- $X$is β-connected space.
- $X$is b-connected space.
- $X$is pre-connected space.
- $X$is α-connected space.
- $X$is connected space.

Proof. From theorem (2.18), in strongly hyperconnected space open sets ands-dense sets are coincide, therefore from corollary (2.15) s-dense sets, β-open sets, b-open sets, pre-open sets, α-open sets and open sets are all coincide, that complete the prove.

CONCLUSION

SD-separated sets and SD-connected spaces have been defined in this article, using the notion of somewhere dense sets. We investigate the properties of the class SD-connected spaces, and study their subspaces behavior and their images by some particular maps. Additionally, we illustrate the relationships between this space with some other known spaces, as strongly hyperconnected space, hyperconnected space and connected space.
Our results are summarized in the following points:

- Separated sets ⟹ SD-separated sets.
- Any space that has a non-empty proper regular open (regular closed) subset is SD-disconnected.
- Any ST_1-space that has more than one element is SD-disconnected.
- Any subset of SD-connected space is SD-connected set.
- Any open subspace of SD-connected space is SD-connected space.
- If Y is an open SD-connected set in a space X, then the relative topology on Y is SD-connected space.
- If Y is a closedSD-connected subspace of a space X, then Y is SD-connected set.
- Image of SD-connected space by SD-irresolute map is SD-connected set.
- Image of SD-connected space by ontoSD-irresolute map is SD-connected space.
- Image of SD-connected space by SD-irresolute map $F: X \to Y$is SD-connected space; where $F(X)$ is open subspace of Y.
- Image of SD-connected space by SD-continuous map is connected space.
- A space X is SD-connected iff every SD-continuous map $F: X \to \{0,1\}$ is constant.
- In SD-connected space; somewhere dense setsand dense sets are coincide.
- Strongly hyperconnectedspace⟹ SD-Connected space⟹Hyperconnected space⟹Connected space.

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